



An excursion through mathematics and its history (the middle ages and beyond)(and some trivia)

MATH DAY 2018—TEAM COMPETITION

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A quick review of the rules

- History (or trivia) questions alternate with math questions
- Math questions are numbered by MQ₁, MQ₂, etc. History questions by HQ₁, HQ₂, etc.
- Math answers should be written on the appropriate sheet of the math answers booklet.
- The answer to every math question will be either an integer (mostly positive) or the square root of a positive integer.
- Square roots of positive integers MUST be entered in the form $m\sqrt{n}$ where m , n are positive integers and n is square free. Examples: $\sqrt{12}$ should be entered as $2\sqrt{3}$, $\sqrt{50}$ should be entered as $5\sqrt{2}$, $3\sqrt{98}$ as $21\sqrt{7}$.
- History questions are multiple choice, answered using the clicker.
- Math questions are worth the number of points shown on the screen when the runner gets your answer sheet. That equals the number of minutes left to answer the question.
- Have one team member control the clicker, another one the math answers booklet

Rules -- Continued

- All history/trivia questions are worth 1 point.
- The team with the highest math score is considered first. Next comes the team with the highest overall score, from a school different from the school of the winning math team. Finally, the team with the highest history score from the remaining schools.

HQ0-Warm Up, no points

Descartes famous phrase *Cogito, ergo sum*, translates as

- A. I think I am a man
- B. I think therefore I am
- C. I stink therefore I am
- D. I think my name is Sam
- E. I drink and eat the ham



René Descartes (Cartesius)
(1596-1650)

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20 seconds

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Time's Up!

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Demonstrating the points system

- For math questions there will be a number in the lower right corner. It will change every minute. Here I am illustrating with numbers changing every 10 seconds. Try to imagine 10 seconds is a minute. The first number tells you the maximum number of points you can get for the question. Assume a question is on the screen.
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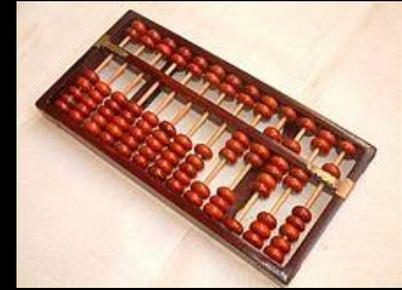
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I may/will give less than the allotted time if I see that all teams have answered.

TIME'S UP!



THE CHALLENGE BEGINS

VERY IMPORTANT!

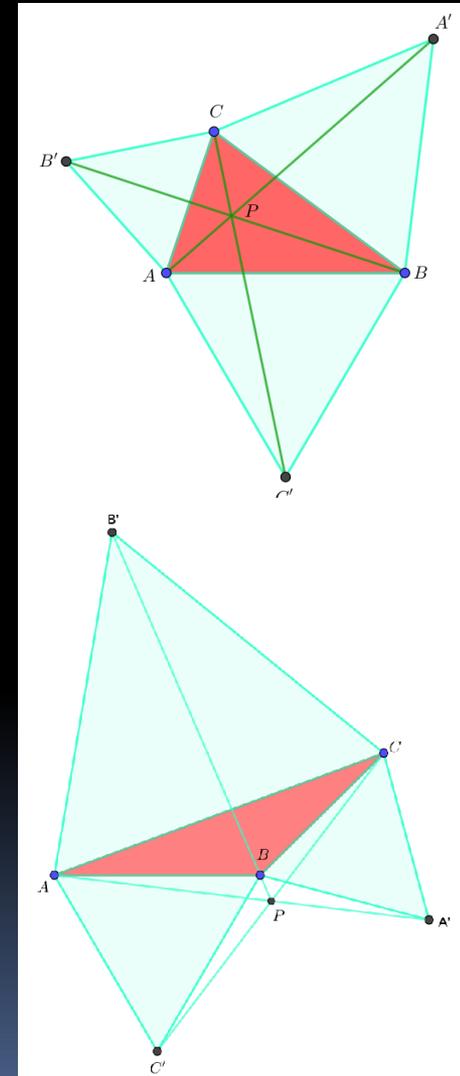
Put away all electronic devices; including calculators.
Mechanical devices invented more than a hundred years ago,
are OK.



HQ1. Isogonic Centers

The isogonic center of a triangle is a point such that the sum of its distances to the vertices of the triangle is minimum. The first person to solve the problem of finding the isogonic center of triangles was

- A. Pierre Fermat
- B. Vincenzo Viviani (1622-1703)
- C. Galileo Galilei (1564-1642)
- D. Evangelista Torricelli (1608-1647)
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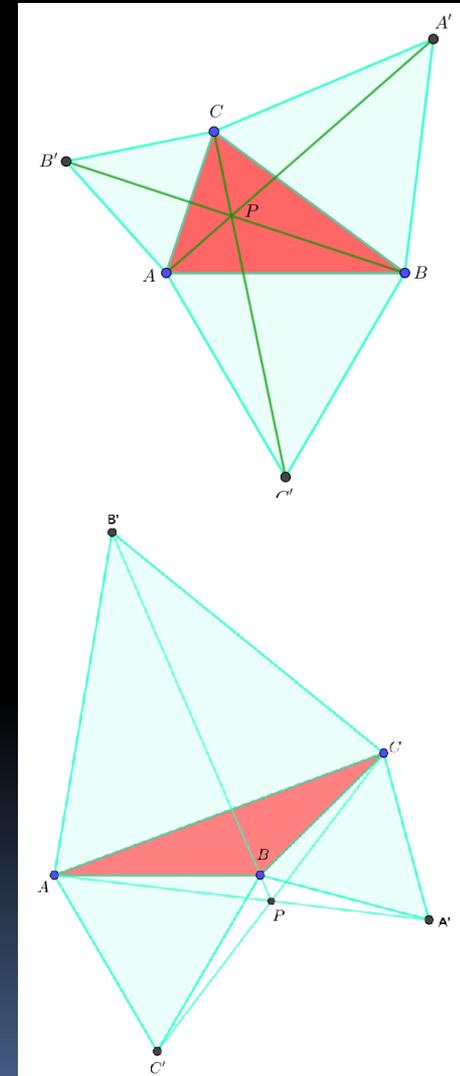


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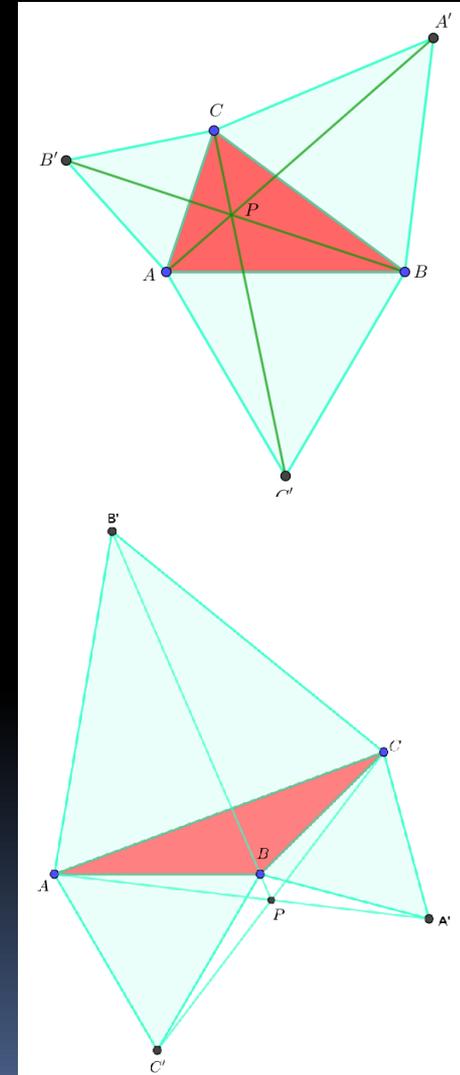


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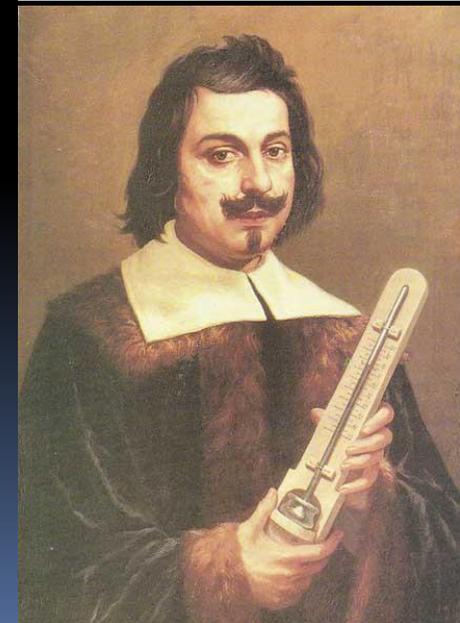
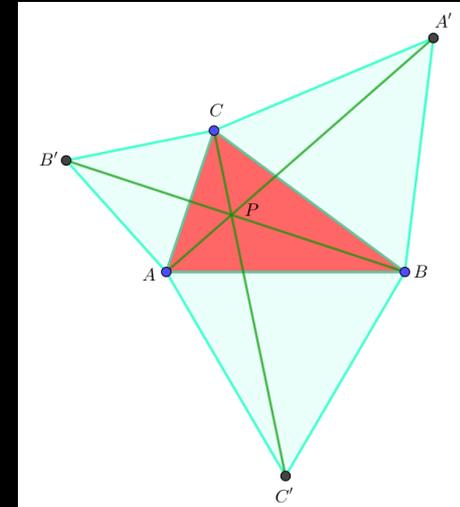
Time's Up!



HQ1. Isogonic Centers

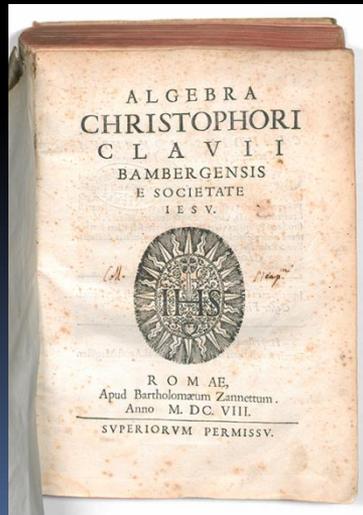
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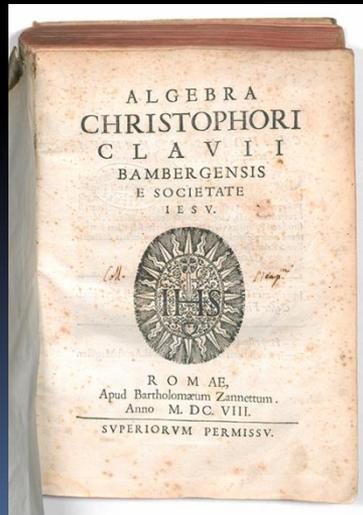
MQ1. A Problem from Clavius

- A father agrees to pay his son 8 cents for every problem correctly solved and to fine him 5 cents for each incorrect solution. At the end of 26 problems neither owes anything to the other. How many problems did the boy solve correctly?



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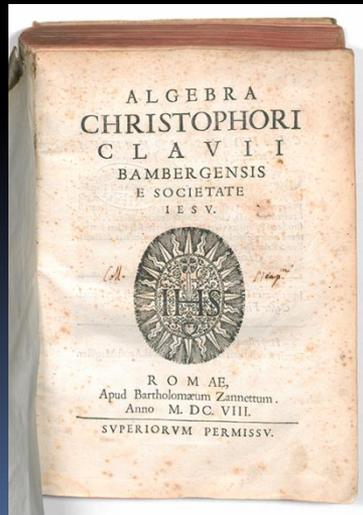
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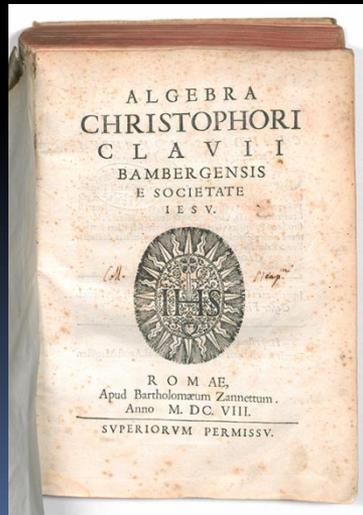


1

TIME'S UP!

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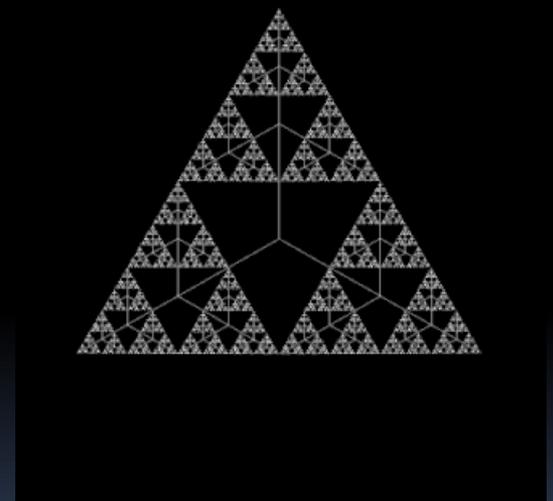
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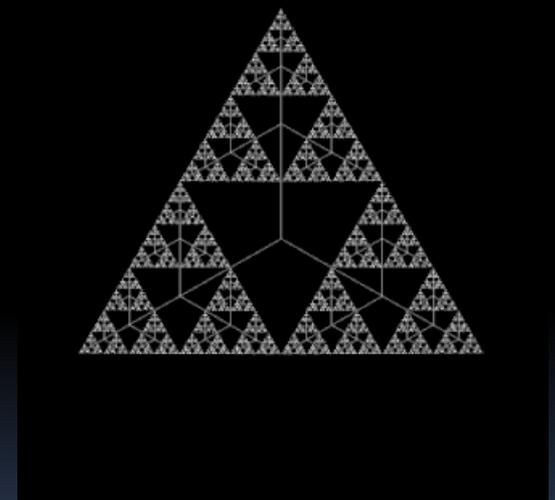
HQ2. The Brachistochrone

- Which of the following phrases best describes the object known as the brachistochrone.
- A. Curve of fastest descent
 - B. Pendulum clock
 - C. Point of maximum curvature
 - D. A primitive telescope
 - E. An ancient geometric drawing implement



HQ2. The Brachistochrone

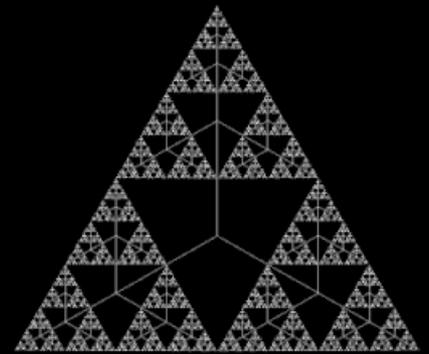
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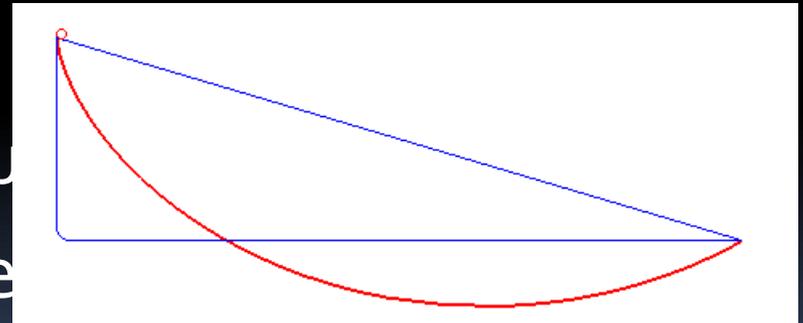
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MQ2. Logarithmic Rhythms

Let $m = \sum_{j=1}^{12} j \ln j$. Then $N = e^m$ is an integer.

What is the highest power of 3 dividing N .

That is, your answer should be the number k such that 3^k divides N , but 3^{k+1} doesn't.



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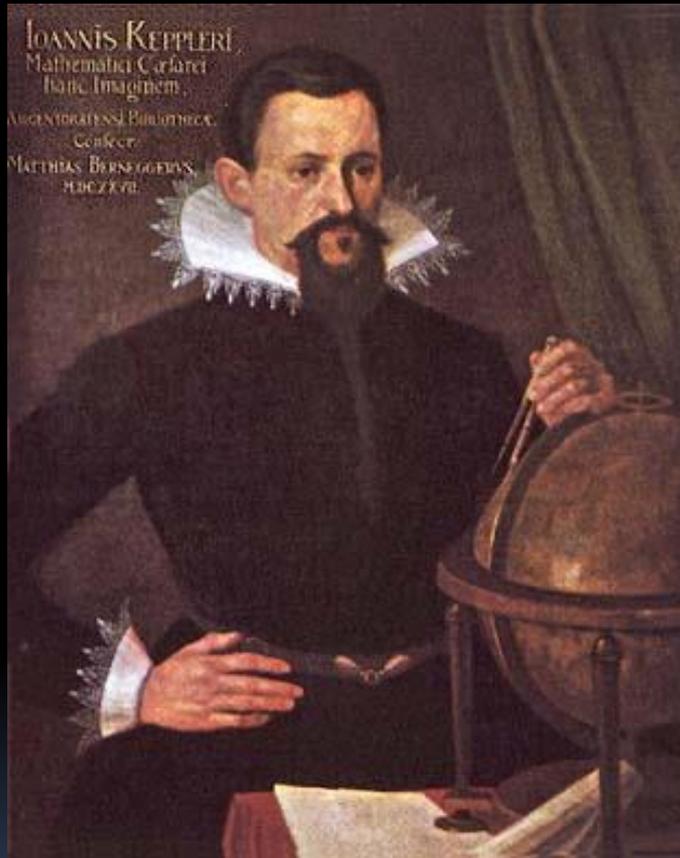
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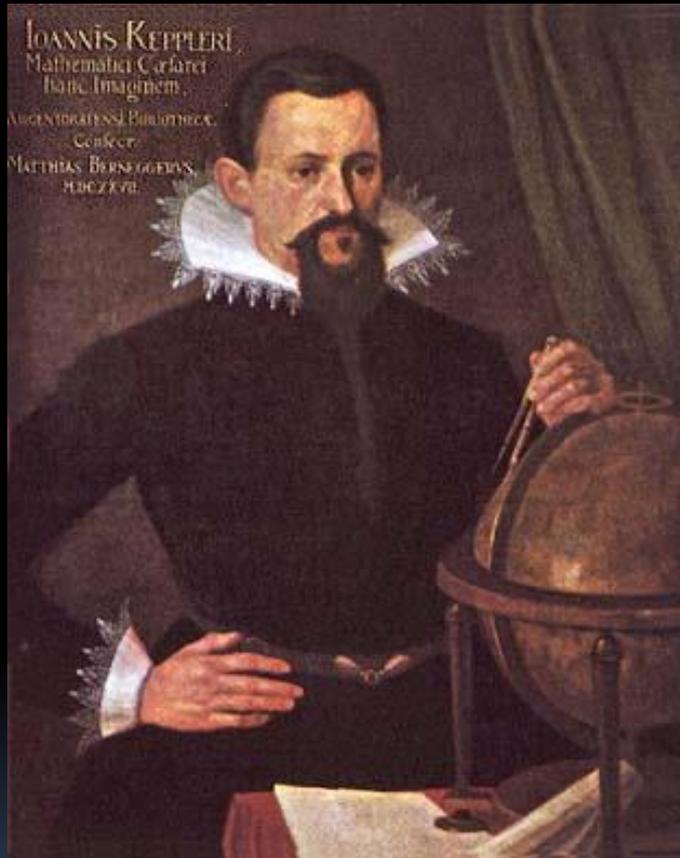
HQ3. Johannes Kepler



Johannes Kepler (1571-1630) is known for his discovery of the planetary laws of motion, and establishing the truth of the heliocentric theory. He also wrote a treatise on

- A. Calculating the distance between planets
- B. Calculating the velocity of light
- C. Calculating the volume of wine barrels
- D. Calculating the earth's diameter.
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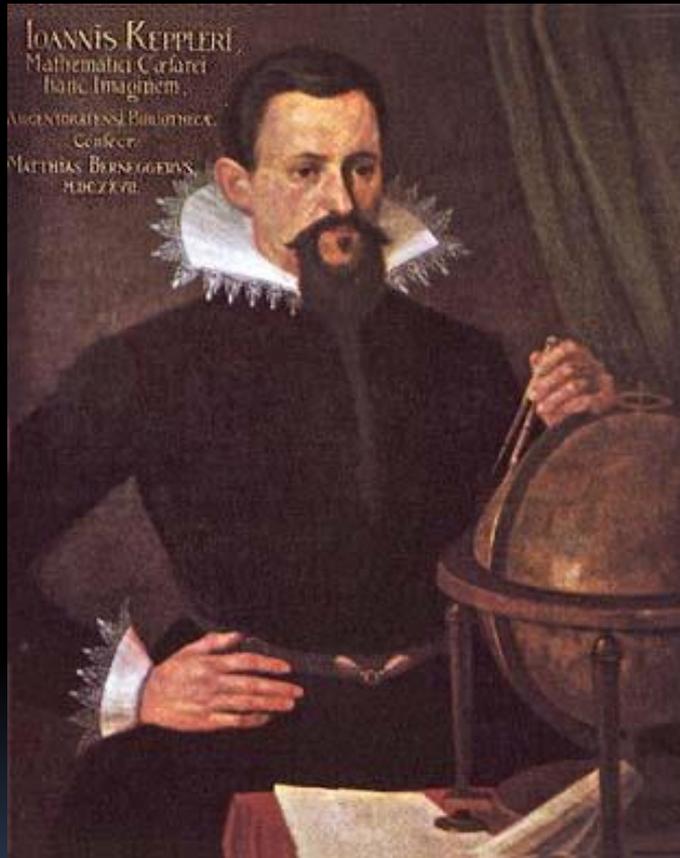


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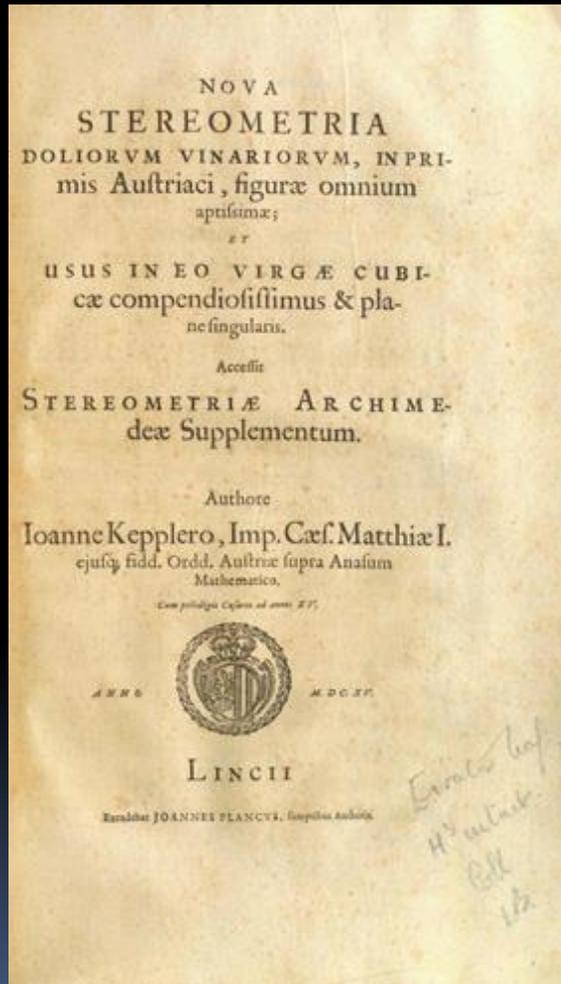


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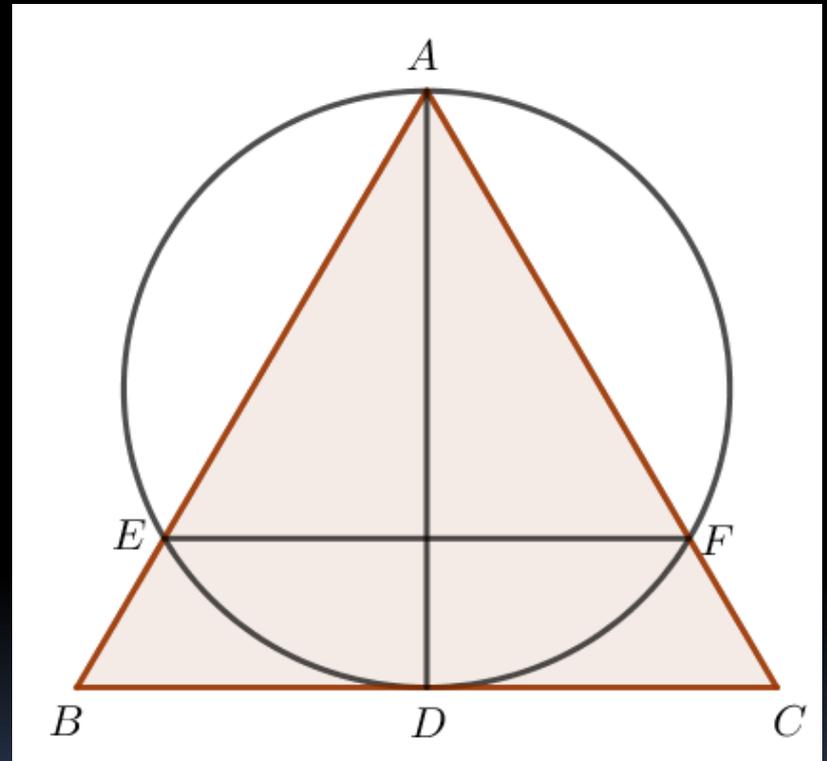
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MQ3. Triangle and Circle

The altitude AD of the equilateral triangle ABC is the diameter of a circle intersecting AB and AC at E and F , respectively. Find the ratio $EF:BC$. Your answer should have the form

$$m:n$$

Where m, n are positive relatively prime integers.

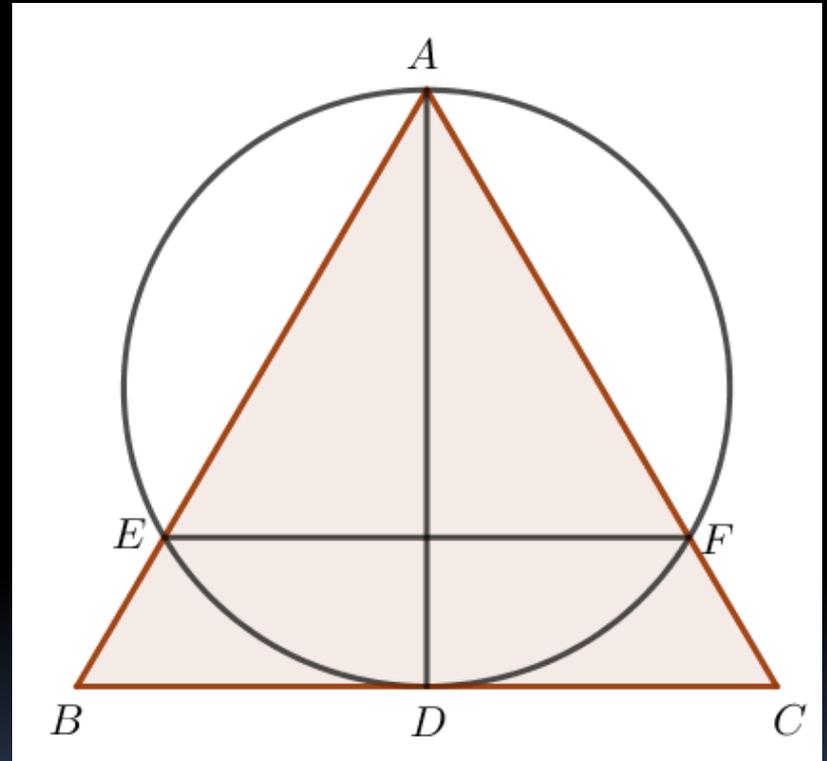


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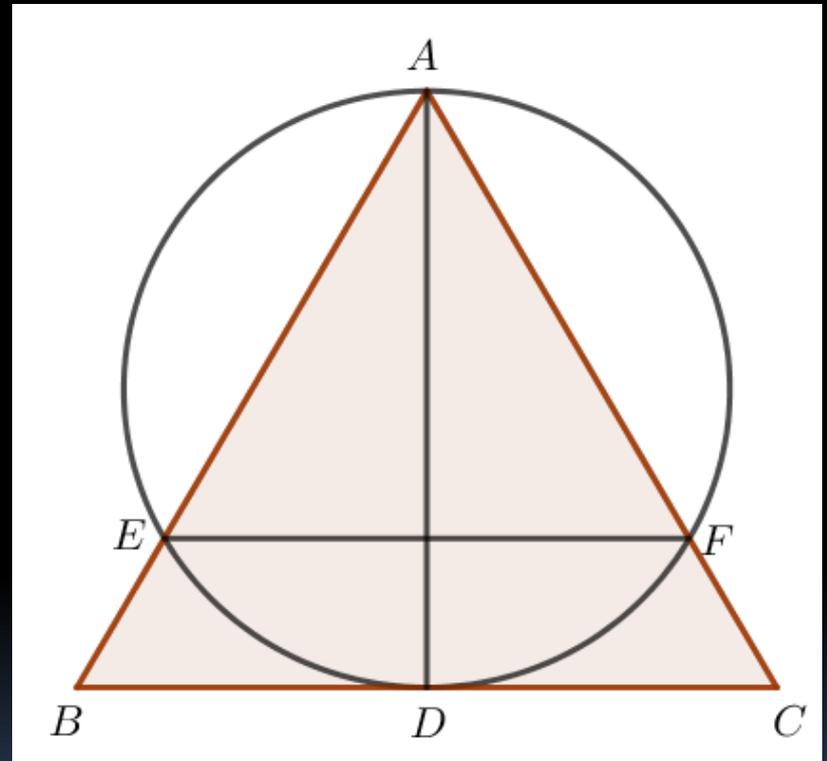
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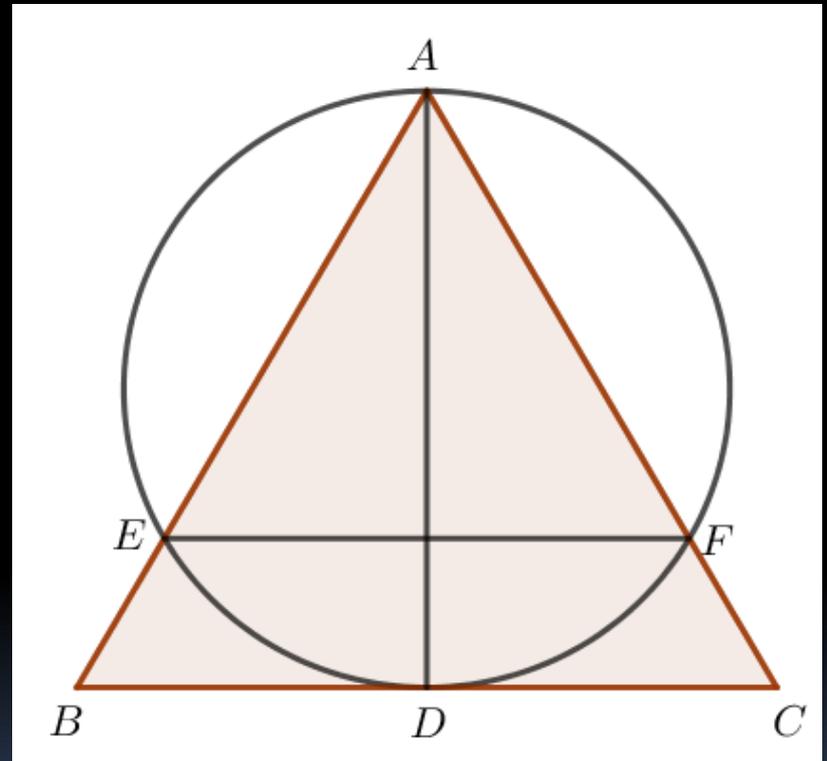


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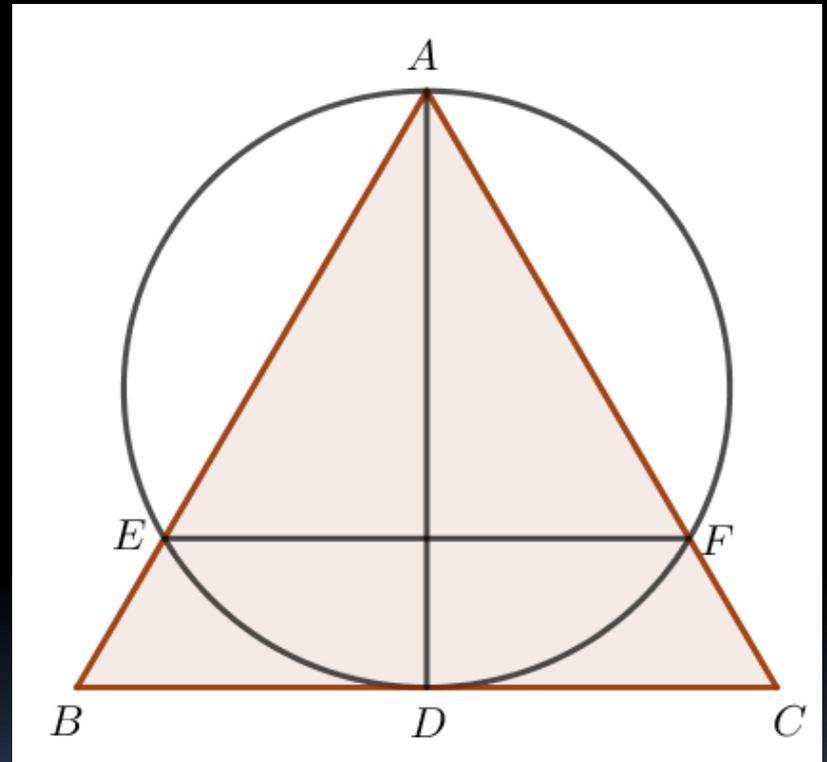
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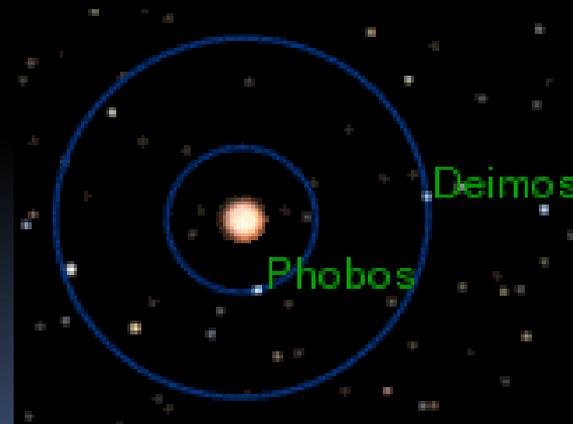


$$EF:BC = 3:4$$

HQ4. Terror and Fear

The satellites of Mars, Deimos and Phobos, were discovered in 1877. However, a book published in 1726 mentions some presumably fictional astronomers who had already discovered that Mars had two satellites and gave orbits not totally inconsistent with the actual orbits. The author of this book was

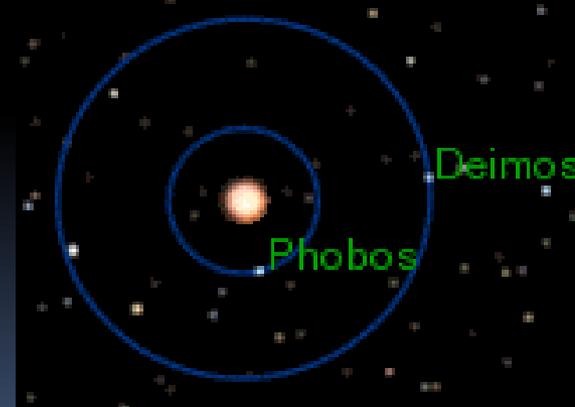
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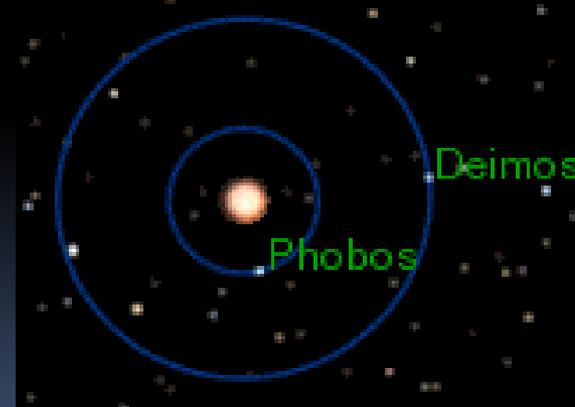


20 Seconds

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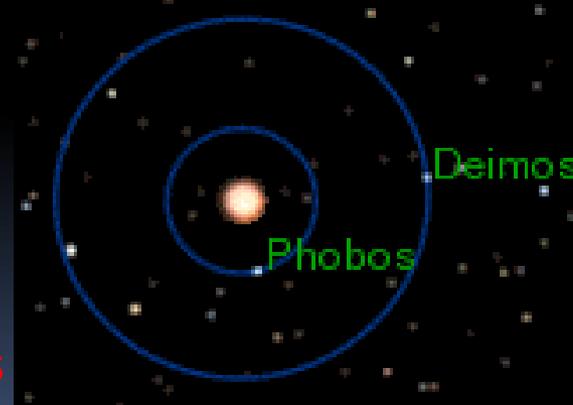


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MQ4. Cryptic Arithmetic

In the following product, A, B, C, D, E, F, G, H are distinct digits.

$$\begin{array}{r} A B C \\ D E \\ \hline F E C \\ D E C \\ \hline H G B C \end{array}$$

What is $HGBC$? Your answer should be a 4 digit integer.



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$$HGBC = 4625$$

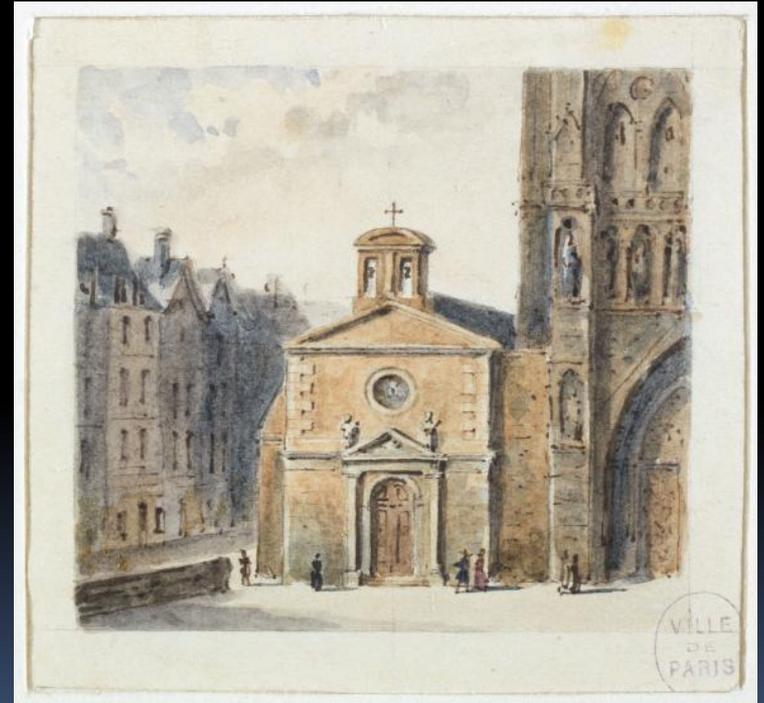
HQ5. The Foundling

- Abandoned by his mother on the doorsteps of a Parisian church, he became one of the most prominent of 18th Century scientists.
- He was
 - A. Joseph-Louis Lagrange
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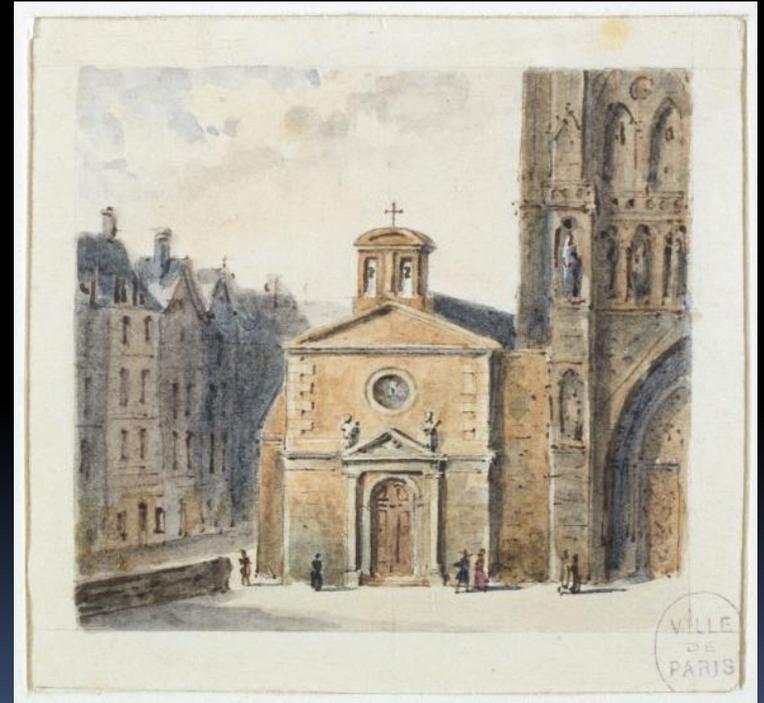
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20 seconds

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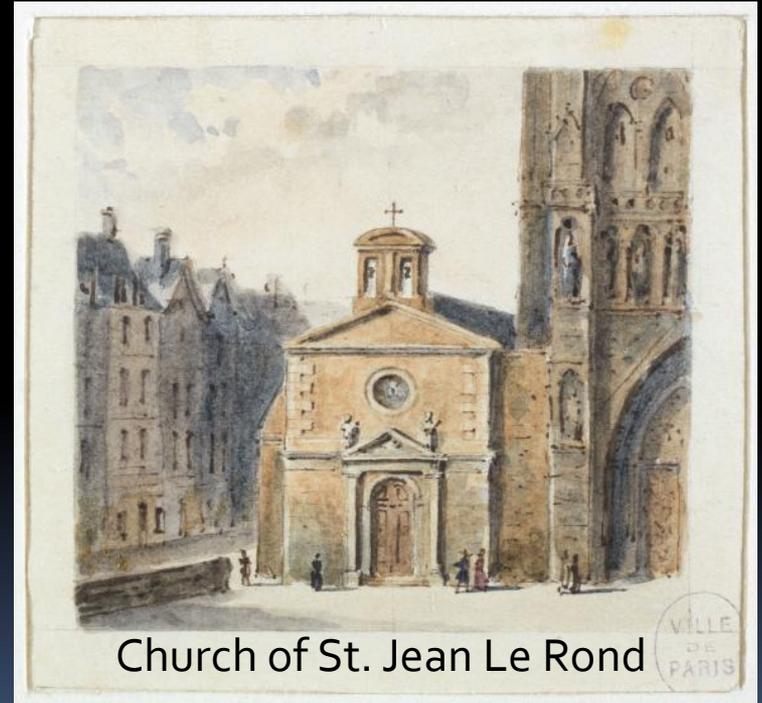
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MQ5. Looking for Primes

For each positive integer n let

$$P(n) = n^4 - 288n^2 + 324$$



Find the sum of ALL values of $P(n)$ that are prime. So, for example, if there are three positive integers for which $P(n)$ equals 3, 5, and 7, respectively, and it is never again prime, then your answer should be 15.

Hint: The answer is NOT 15.

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Find the sum of ALL values of $P(n)$ that are prime. So, for example, if there are three positive integers for which $P(n)$ equals 3, 5, and 7, respectively, and it is never again prime, then your answer should be 15.

$$f(1) + f(17) = 37 + 613 = 650$$

HQ6. Bishop Berkeley



George Berkeley, Bishop of Cloyne (1685-1753) was an early critic of Calculus. He called the derivative

- A. A poorly defined concept
- B. An idea whose time will never come
- C. A hope unfulfilled
- D. The ghost of a departed quantity
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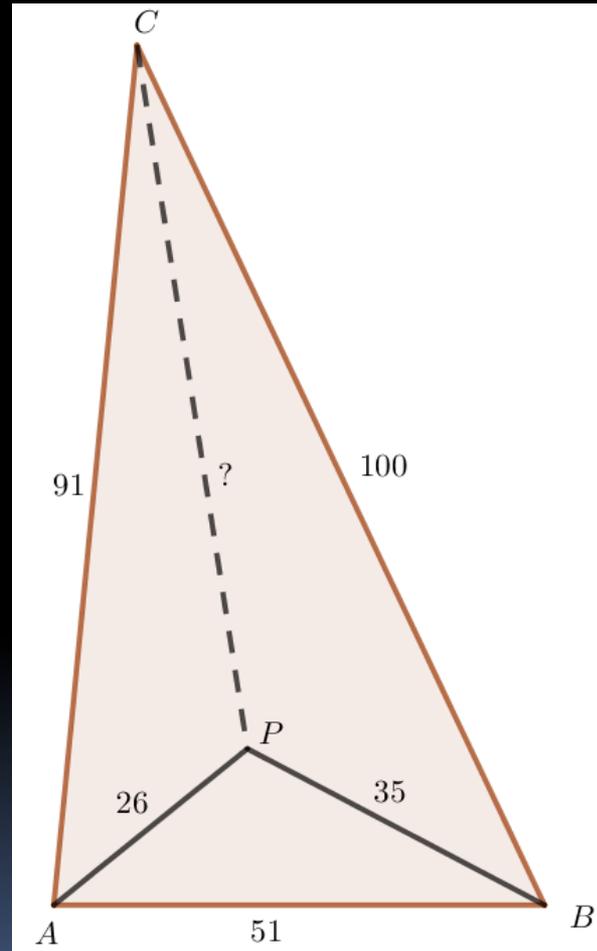


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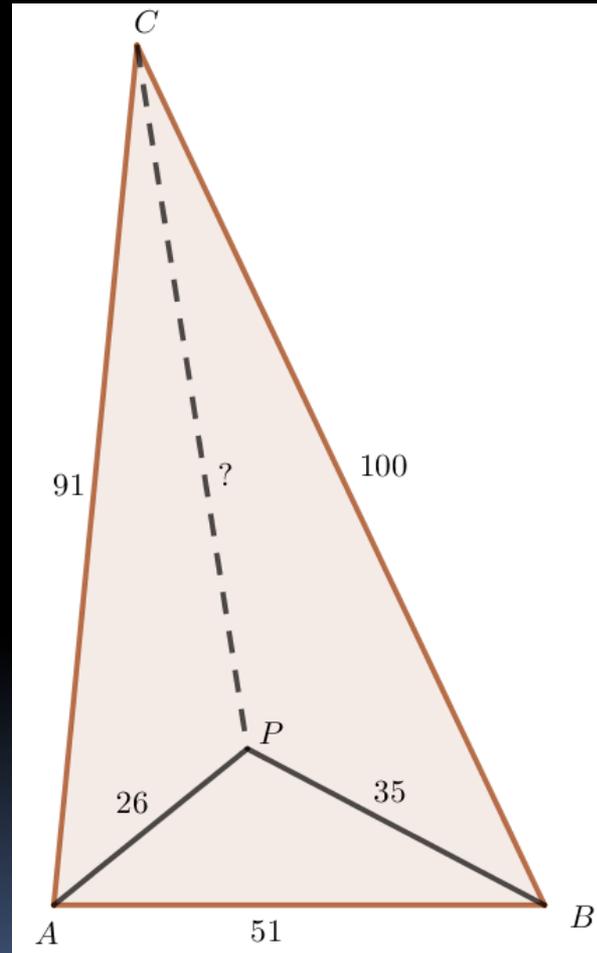
MQ6. A Tricky Triangle

Triangle ABC has side lengths $BC = 100$, $CA = 91$ and $AB = 51$. An interior point P is such that $AP = 26$ and $BP = 35$. Calculate CP .



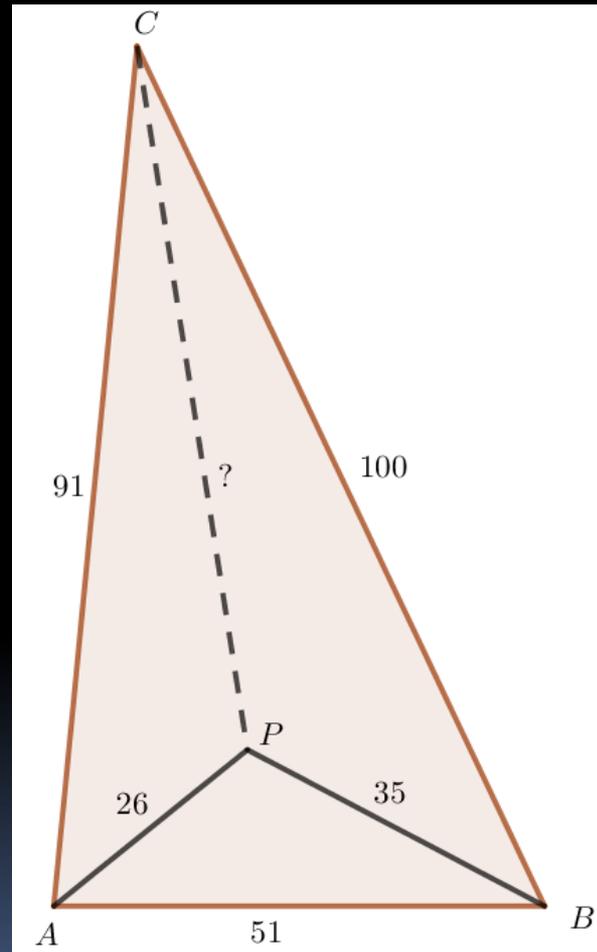
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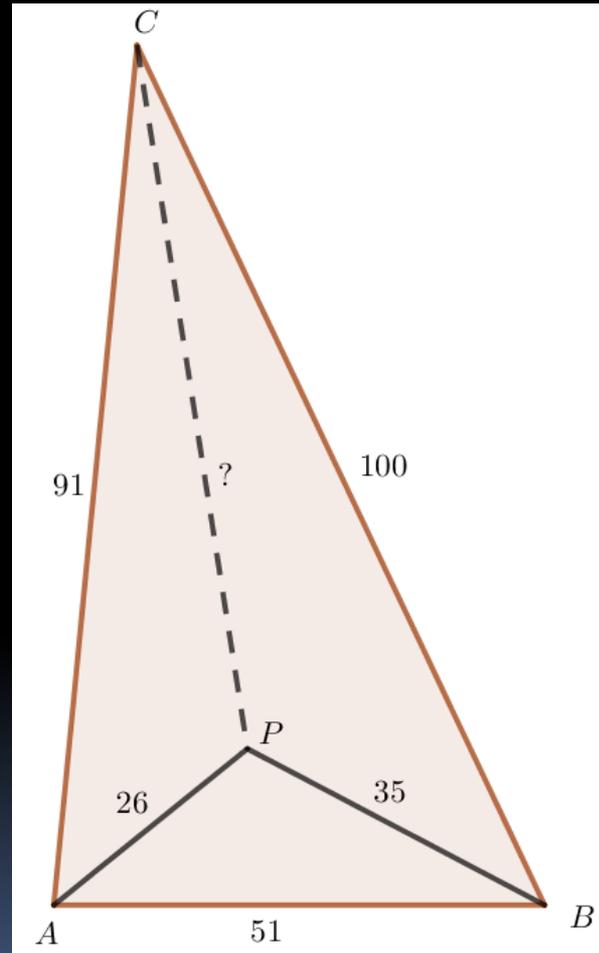
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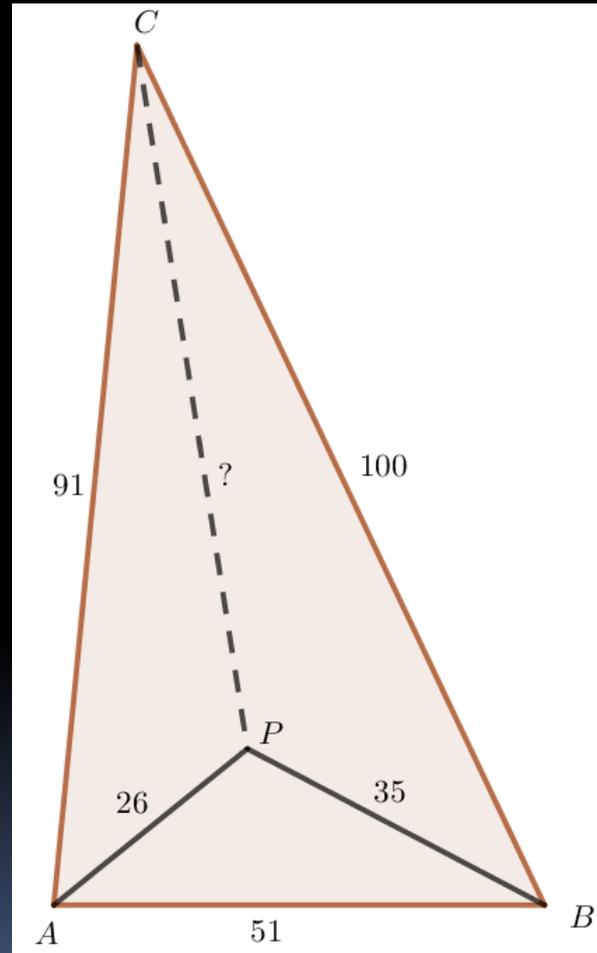
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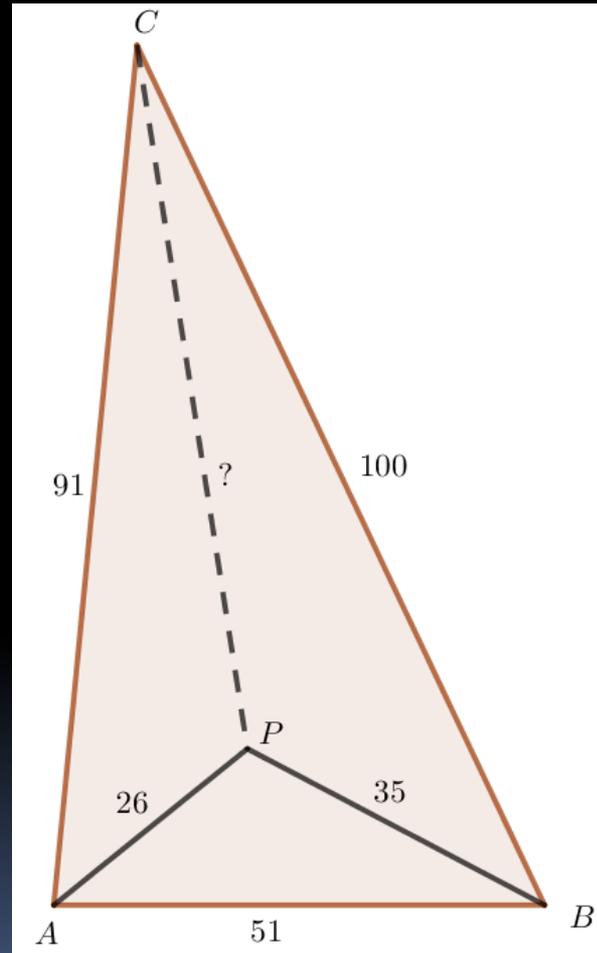
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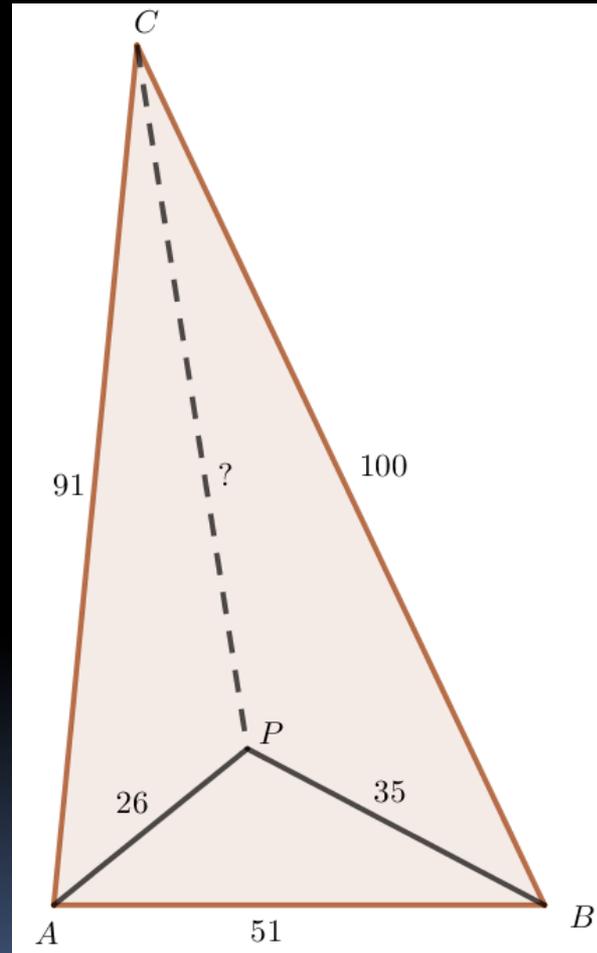


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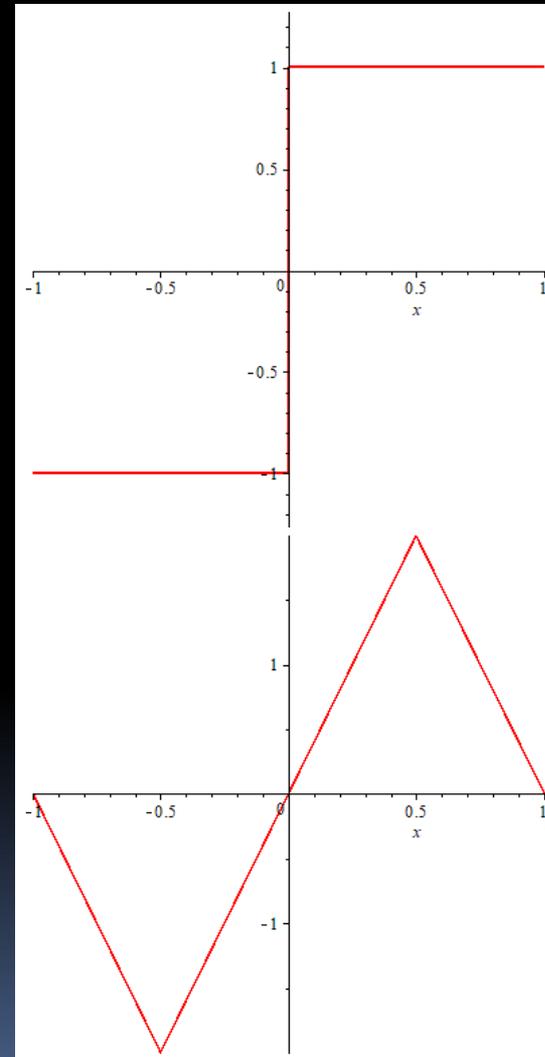
$$CP = 75$$



HQ7. Trigonometric Sums

The discovery by Joseph Fourier (1768-1830) that all periodic functions can be approximated by sums of sines and cosines marks a milestone in mathematics and its applications. He published his findings in a book called

- A. Angular Disquisitions
- B. The Analytic Theory of Heat
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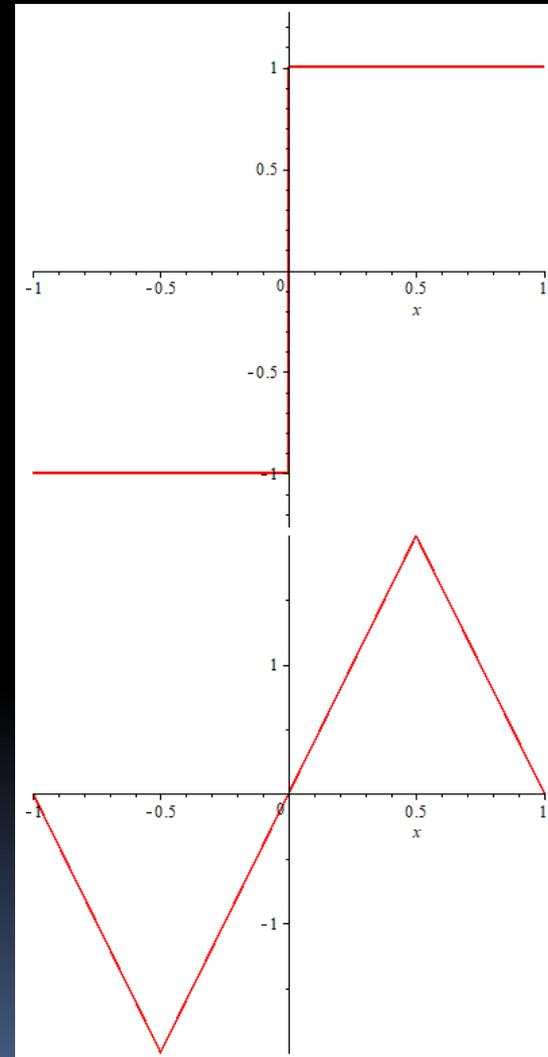


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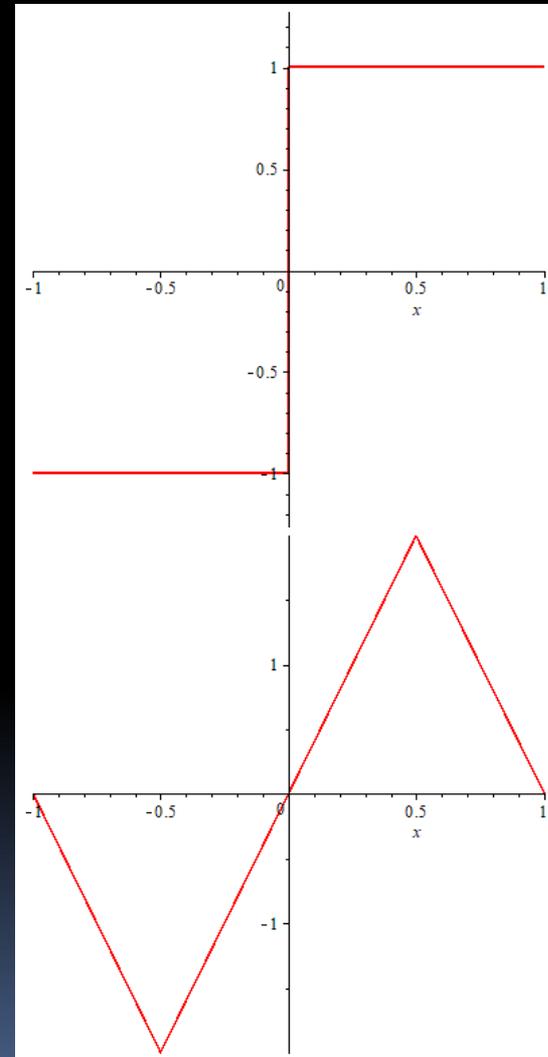


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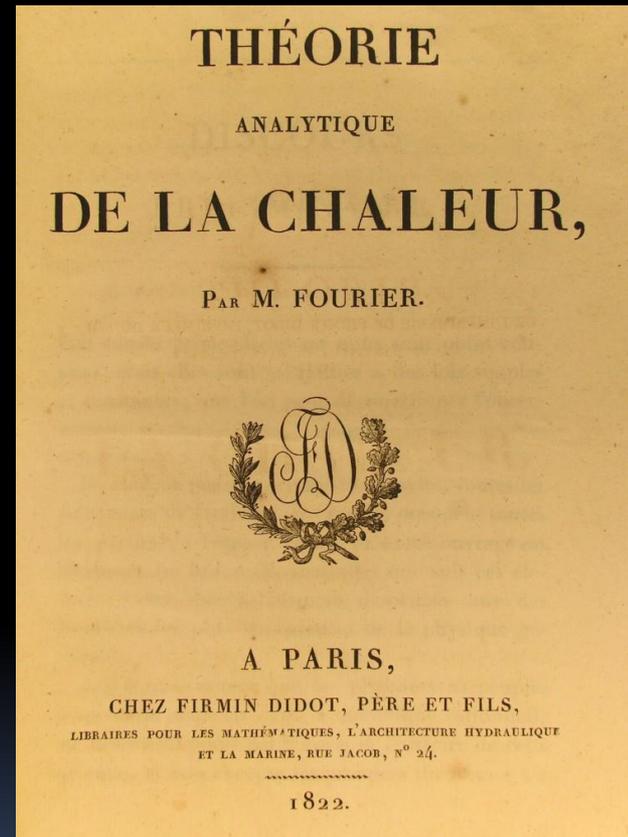
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MQ7. Square Roots

Evaluate $\sum_{k=1}^{2018} \lfloor \sqrt{k} \rfloor$.

(If x is a real number then $\lfloor x \rfloor$ is the largest integer m such that $m \leq x$.)



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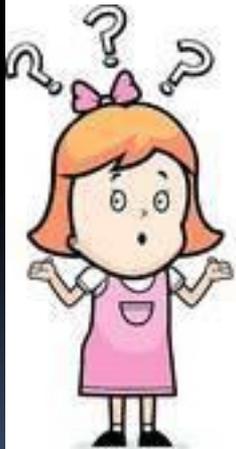
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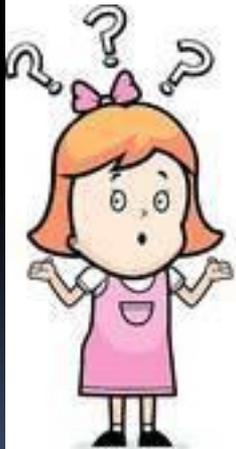
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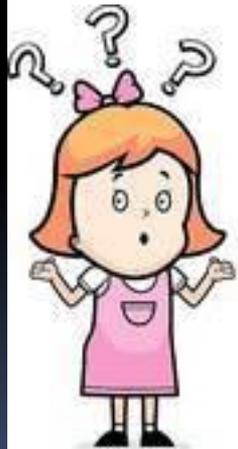
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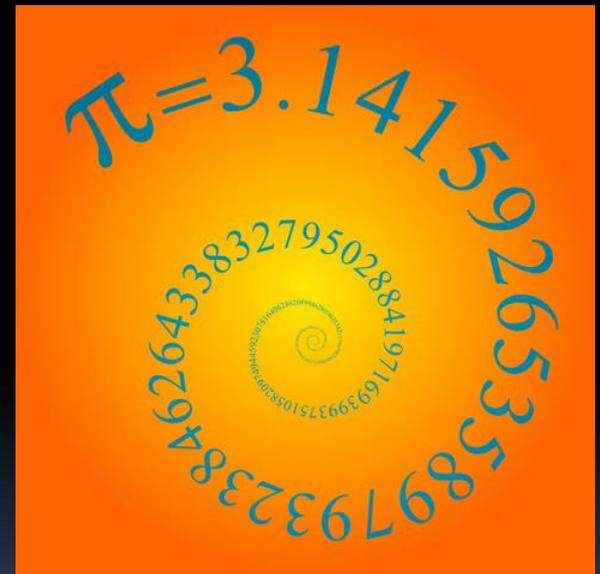
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59466

HQ8. Finding π

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 - A. François Viète in 1579
 - B. Ludolph van Ceulen in 1610
 - C. John Wallis in 1650
 - D. James Gregory in 1671
 - E. Abraham Sharp in 1699



Time's Up!

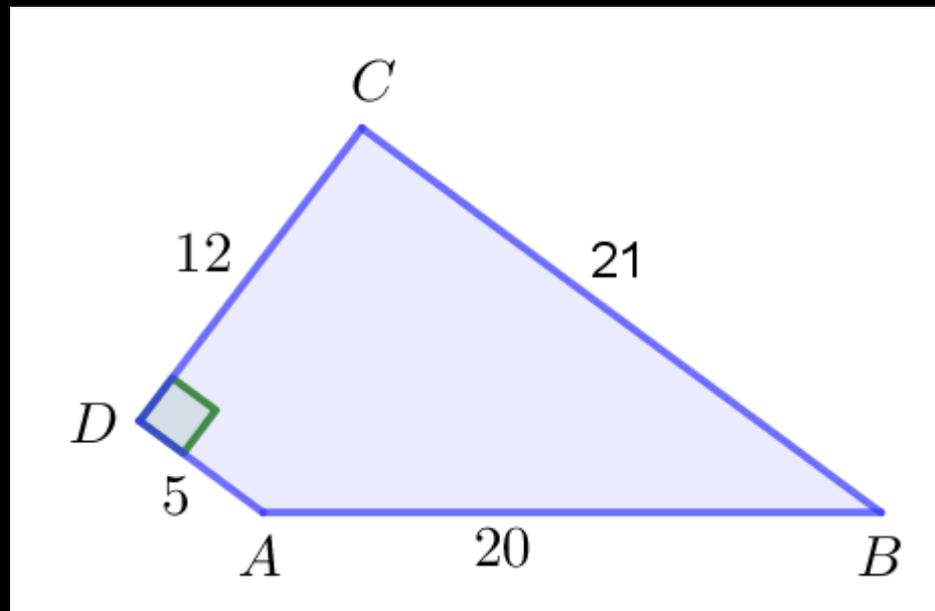
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MQ8. Quadrilateral Quirks

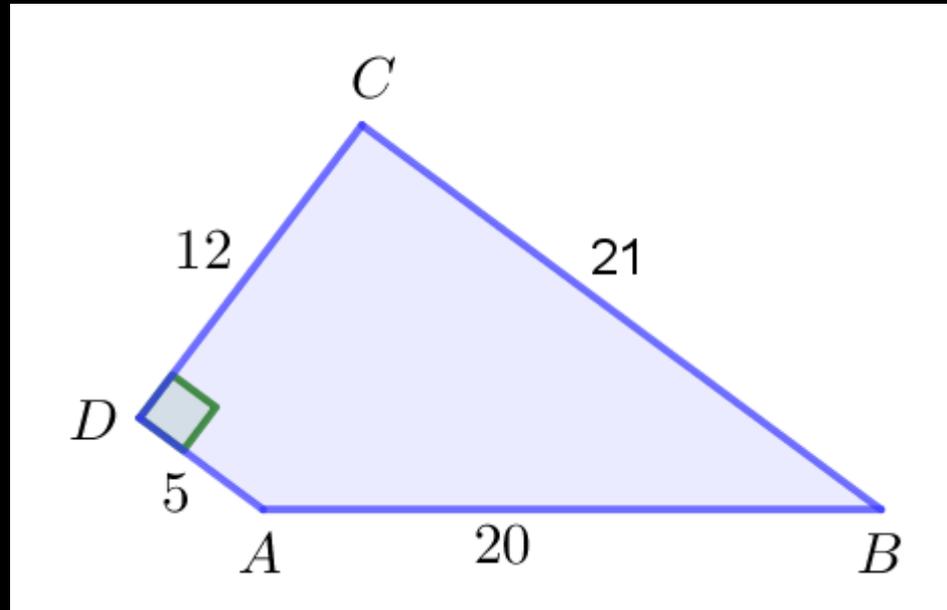
The quadrilateral on the right has sides $AB = 20$, $BC = 21$, $CD = 12$, and $DA = 5$. The angle at D is a right angle.



Find its area.

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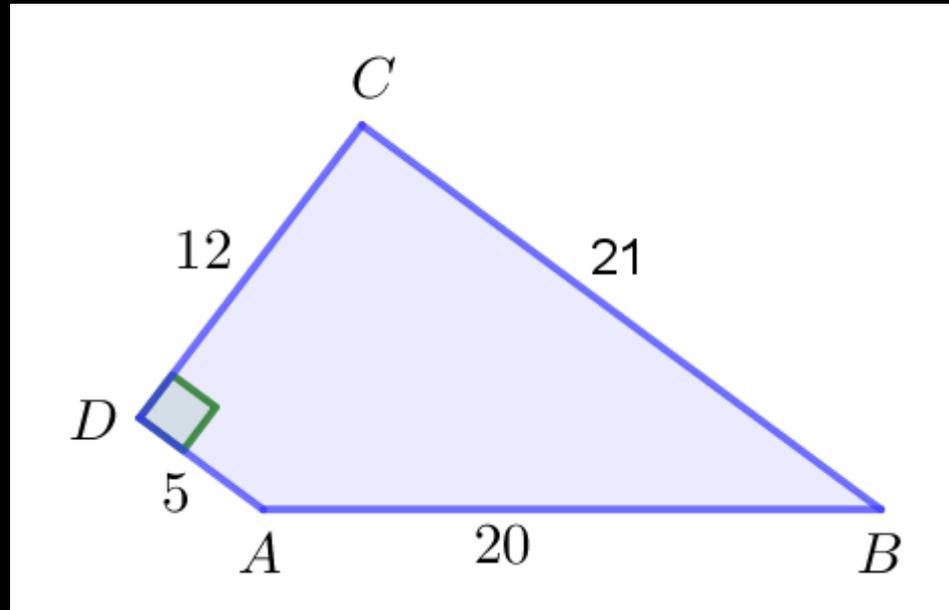
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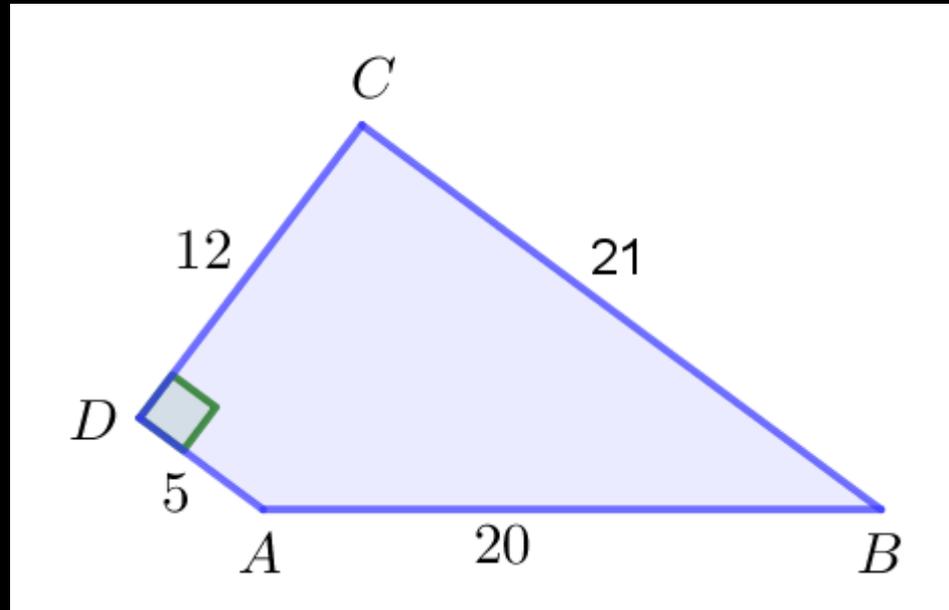
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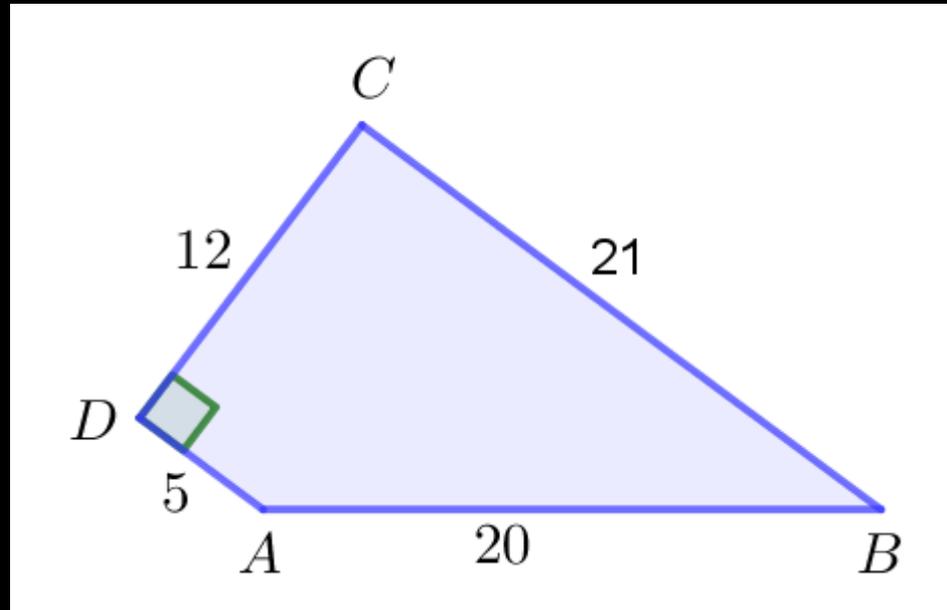
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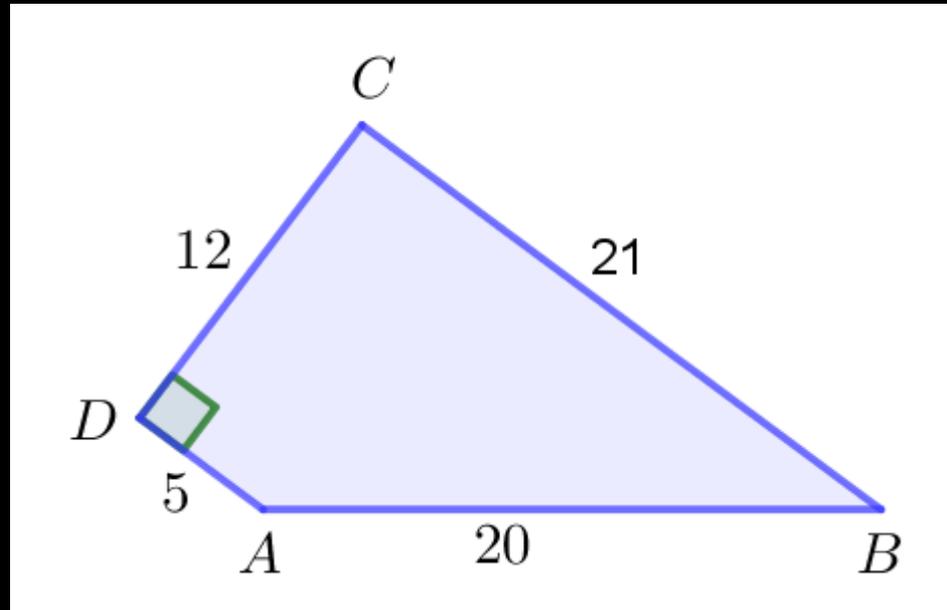


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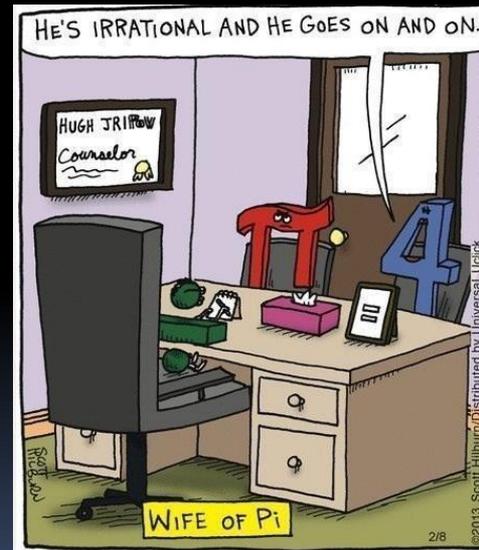
Find its area.

$$[ABCD] = 156$$

HQ9. The Irrationality of π

- Except for $x = 0$, x (in radians) and $\tan x$ cannot both be rational. This Theorem, proved in 1768 establishes, for the first time, that $\frac{\pi}{4}$, hence π , is irrational (because $\tan \frac{\pi}{4} = 1$). Its author was

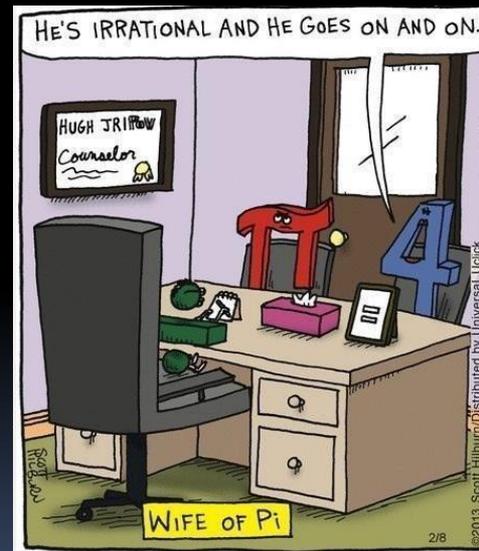
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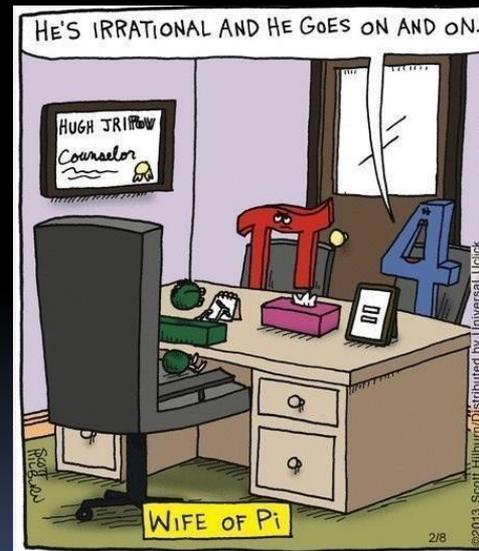


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MQ9. Logarithms Again

- Here $\log z$ denotes the logarithm in base 10 of z .
- Find $x > 1$ such that

$$x^{7 \log 3} = \frac{1}{27} 3^{6(\log x)^2}$$



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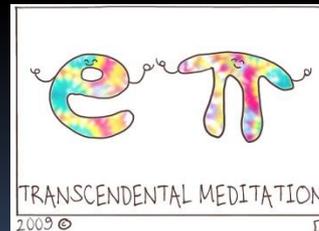
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$$x = 10\sqrt{10}$$

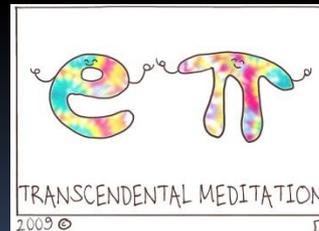
HQ10. Way Beyond Irrational

- In 1882 Ferdinand v. Lindemann proved that π was *transcendental*. A number is transcendental if
 - A. Its decimal expansion is completely random
 - B. All of its powers are irrational
 - C. It cannot be approximated by rational numbers
 - D. It is the zero of a polynomial of degree at least 1 with irrational coefficients.
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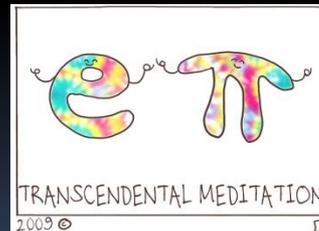
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20 seconds

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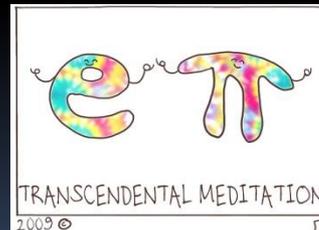
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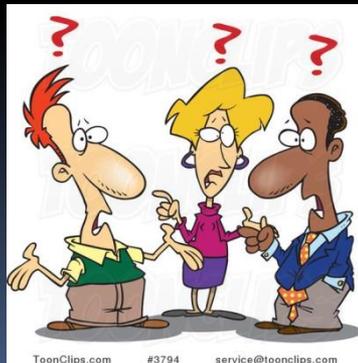
MQ10. Polynomially Correct

- The coefficient of x^{50} when one expands

$$(1 + x^2)^{1000} + x^2(1 + x^2)^{999} + x^4(1 + x^2)^{998} + \dots + x^{1998}(1 + x^2) + x^{2000}$$

and collects terms has the form $\frac{n!}{(n-m)!m!}$ where

$m \geq n/2$. What is m ?



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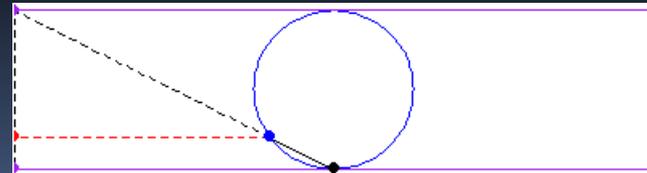
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$$m = 976$$

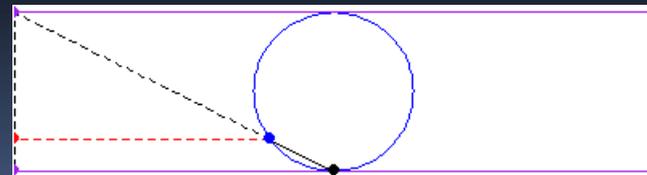
HQ11. An Italian Polymath

- Maria Gaetana Agnesi (1718-1799) was an extraordinarily gifted woman who already at age 9 published an essay in Latin defending higher education for women. Today she is mostly known for her study of a curve mistranslated into English as
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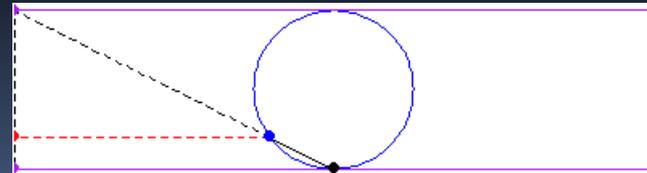
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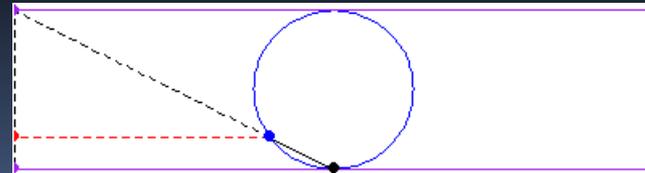


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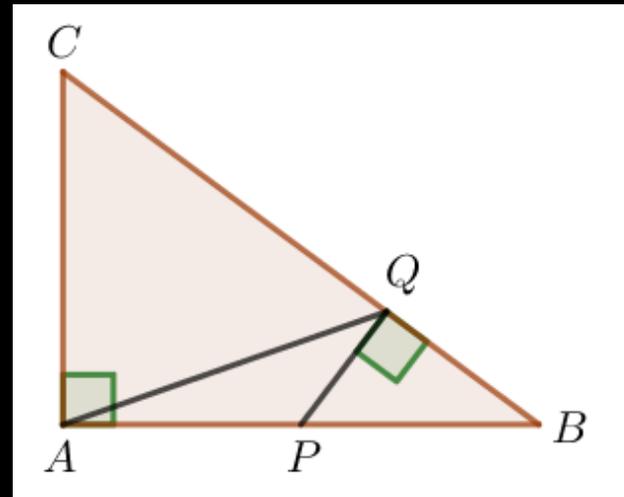


MQ11. You Better Ask Ptolemy

ABC is a right triangle. The point P is placed on side AB in such a way that $AP = PB = 2$. We also have $AC = 3$. The point Q on the hypotenuse of the triangle is such that the segment PQ is perpendicular to the hypotenuse. Find the length of AQ . Your answer should be in the form

$$\frac{m\sqrt{n}}{k}$$

where m, n, k are positive integers, m and k are relatively prime and n is square free.

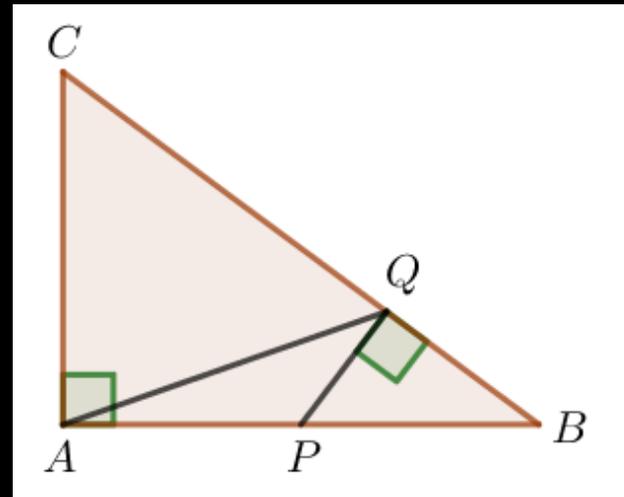


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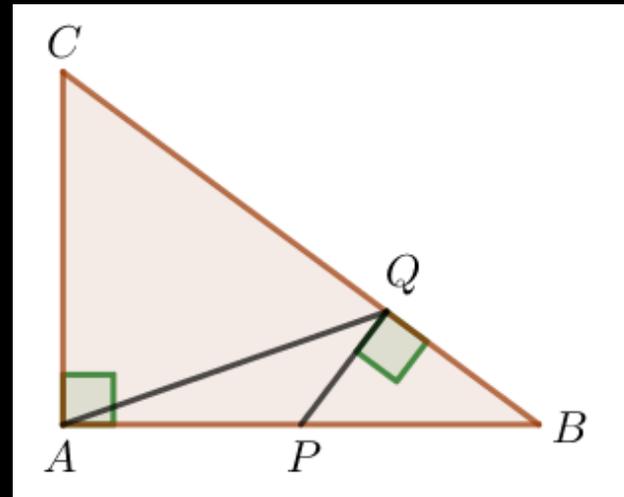


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where m, n, k are positive integers, m and k are relatively prime and n is square free.



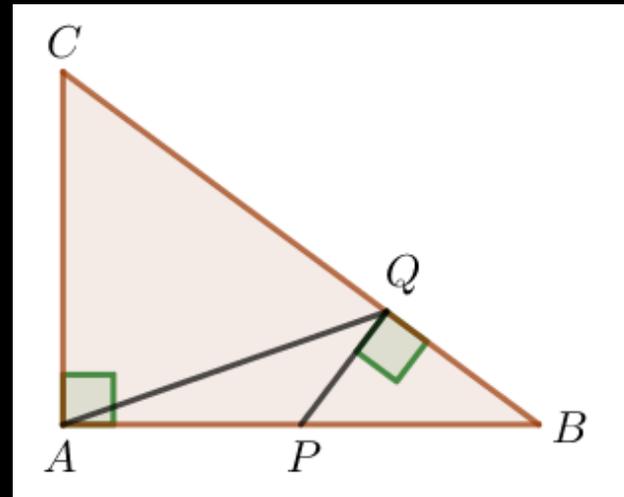
3

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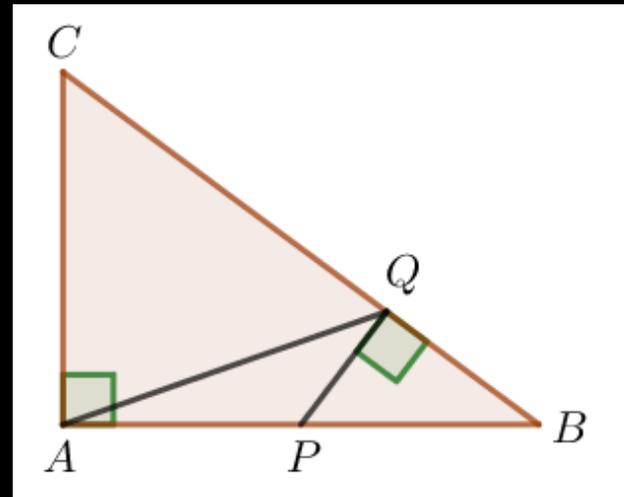


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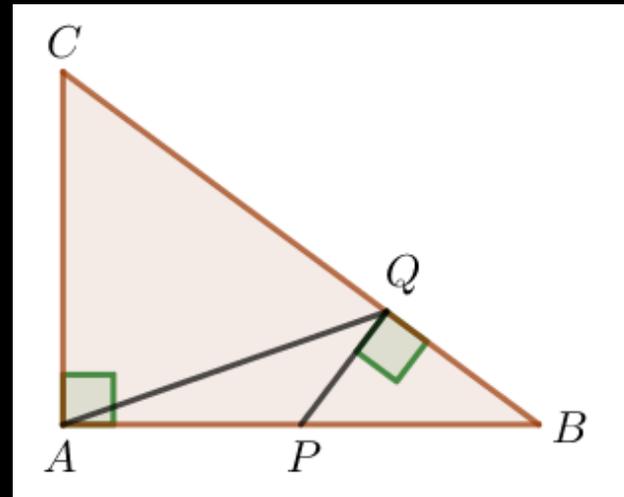
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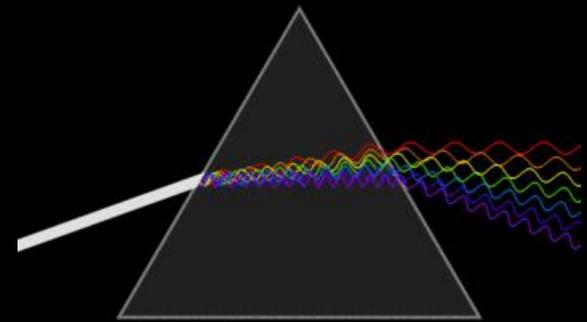
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$$AQ = \frac{4\sqrt{13}}{5}$$

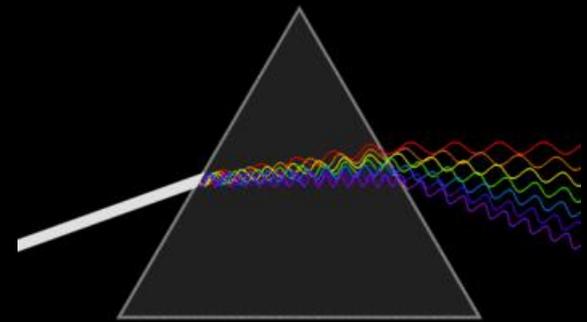
HQ12. The Nature of Light

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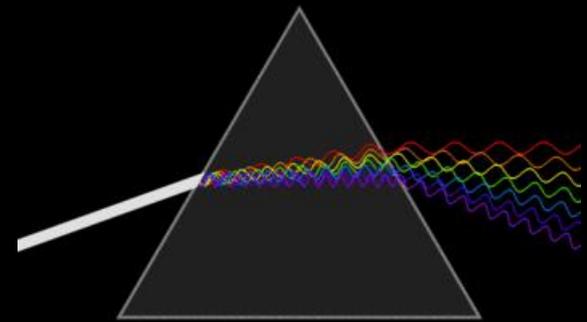
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MQ12. Progressive Geometry

- Suppose we have a geometric progression of 5 integers, all of them less than or equal 100. The sum of the five terms is 211. Let S be the sum of all the terms of the progression that are squares of integers. What is S ?
- (If you decide there is no such progression, or there is one but none of its terms are perfect squares, your answer should be 0)



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$$16, 24, 36, 54, 81$$

$$16 + 24 + 36 + 54 + 81 = 211$$

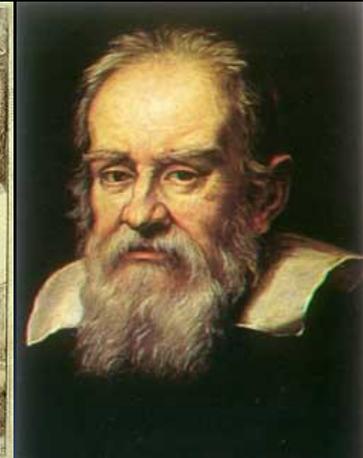
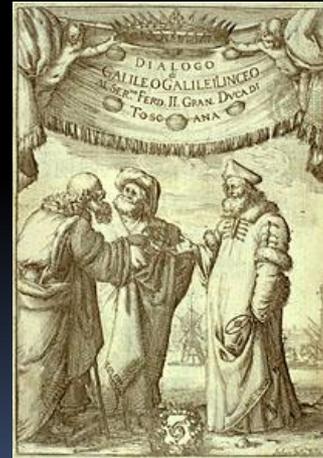
$$16 + 36 + 81 = 133$$

$$S = 133$$

HQ13. Galileo

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- A. 1789
- B. 1801
- C. 1847
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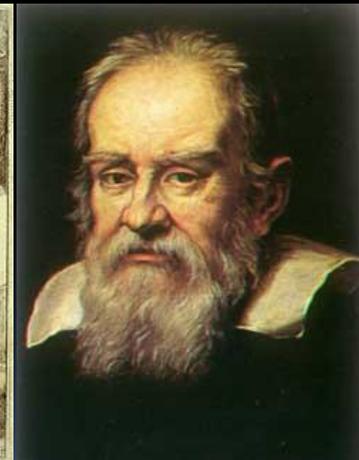
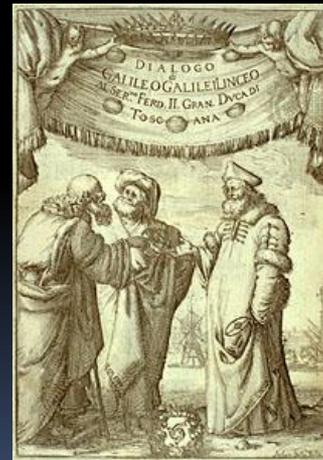


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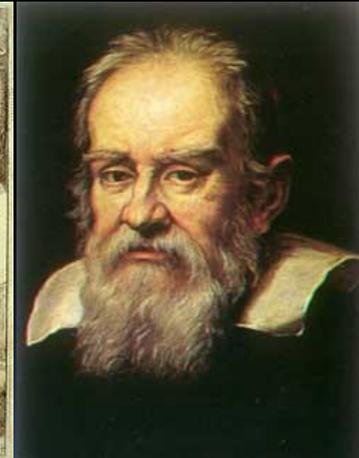
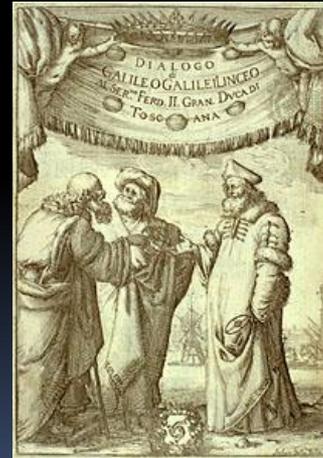


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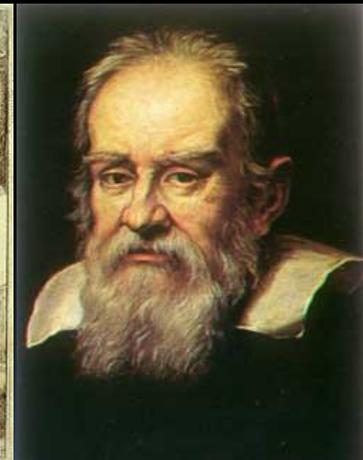
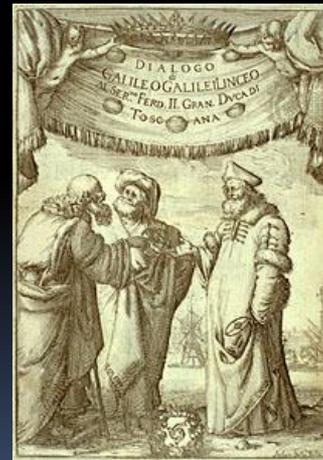
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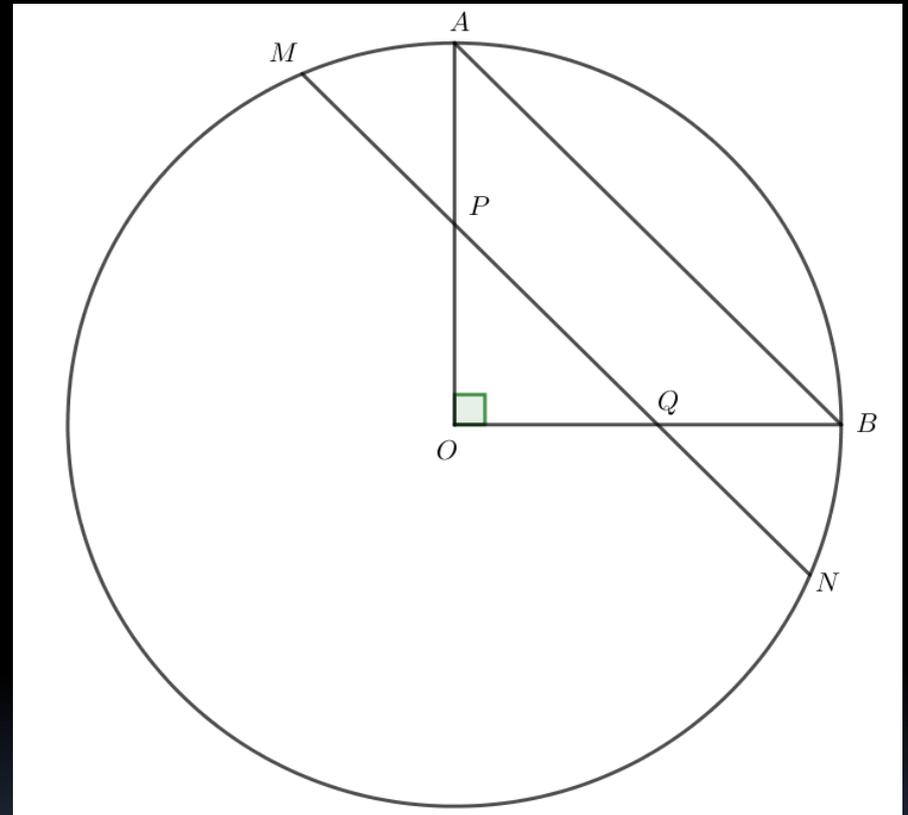
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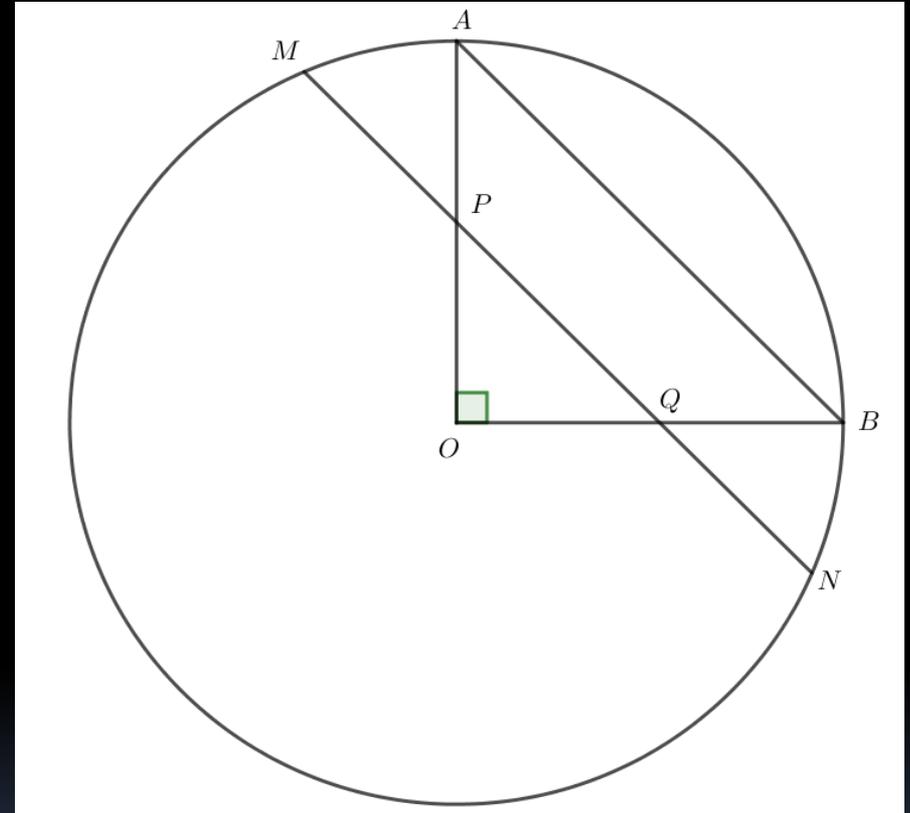
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In the circle pictured on the right, radius OA is perpendicular to radius OB . Segment MN is parallel to AB and intersects OA at P and OB at Q . If $MP = 3$ and $PN = 7$, what is the radius of the circle?



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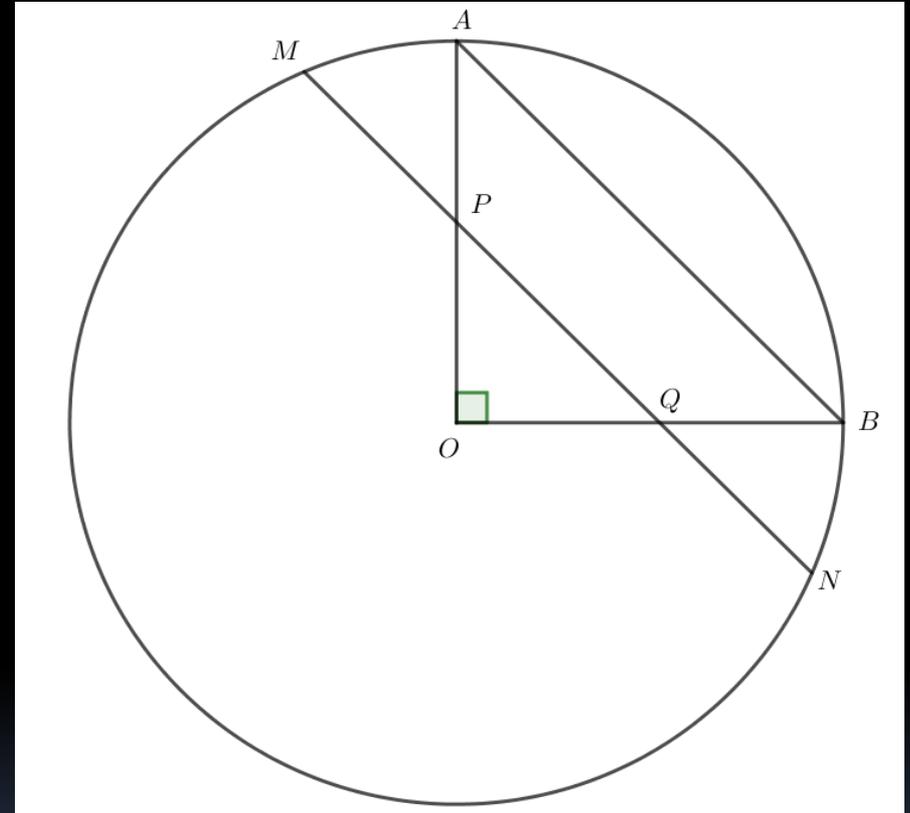
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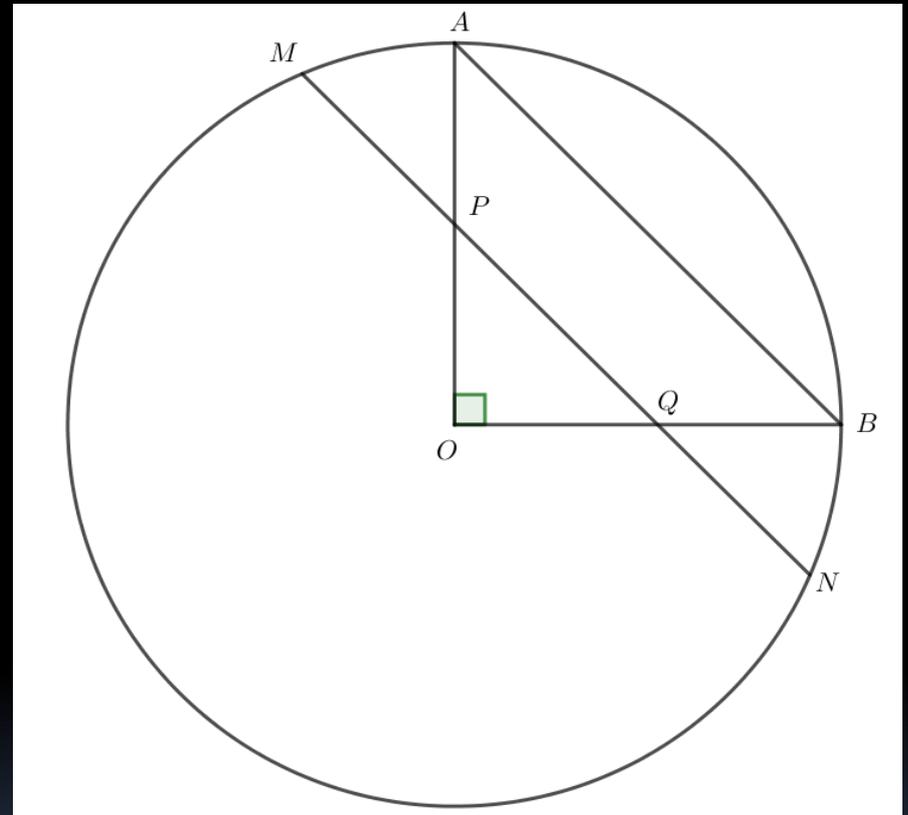
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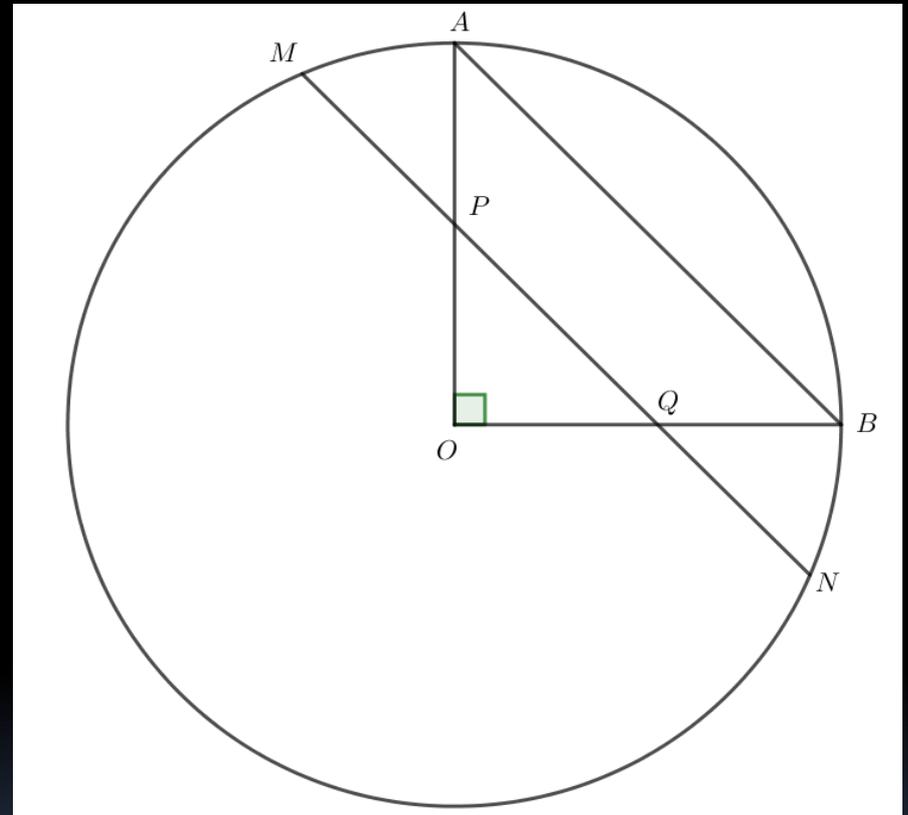
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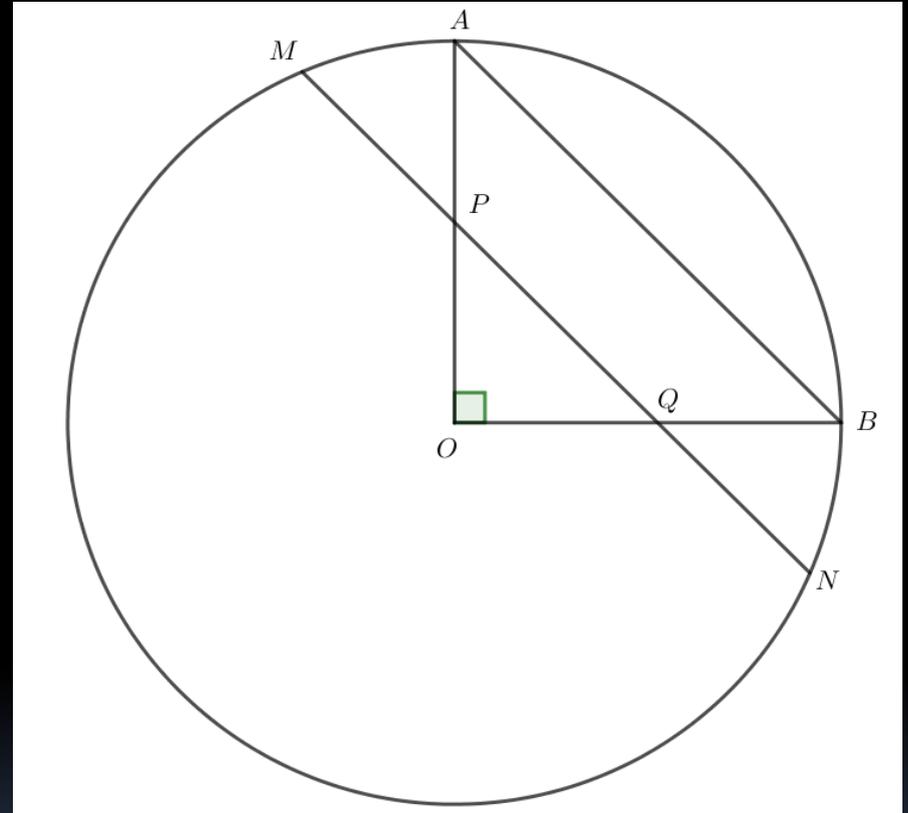


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$$\text{Radius} = \sqrt{29}$$



HQ14. A Great Tragedy

- Born in Iran she won gold more than once at the IMO and was the first woman to be awarded the Fields Medal (in 2012). She died at age 40, a promise partially unfulfilled. She was

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